

WMA11_P1(IAL)_Summer_2019_Q1

Solution

1. Differentiation of the curve equation

To find the **derivative** $\frac{dy}{dx}$, we first rewrite the equation of the curve C in a form suitable for the **power rule**. The given equation is:

$$y = \frac{1}{8}x^3 - \frac{24}{\sqrt{x}} + 1$$

- **Rewrite the terms:** The term involving the square root can be expressed using a negative fractional exponent:

$$\frac{24}{\sqrt{x}} = 24x^{-1/2}$$

Thus, the equation becomes:

$$y = \frac{1}{8}x^3 - 24x^{-1/2} + 1$$

- **Apply the power rule:** Differentiating each term with respect to x :

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1}{8}x^3 \right) - \frac{d}{dx} (24x^{-1/2}) + \frac{d}{dx} (1) \\ &= \left(3 \cdot \frac{1}{8}x^{3-1} \right) - \left(-\frac{1}{2} \cdot 24x^{-1/2-1} \right) + 0 \\ &= \frac{3}{8}x^2 + 12x^{-3/2} \end{aligned}$$

- **Simplify the expression:** Converting the negative exponent back to radical form:

$$\frac{dy}{dx} = \frac{3}{8}x^2 + \frac{12}{x\sqrt{x}}$$

(a) $\boxed{\frac{dy}{dx} = \frac{3}{8}x^2 + \frac{12}{x^{3/2}}}$

2. Finding the equation of the tangent at point $P(4, -3)$

The **tangent** to a curve at a specific point is a straight line with a **gradient** equal to the value of the derivative at that point.

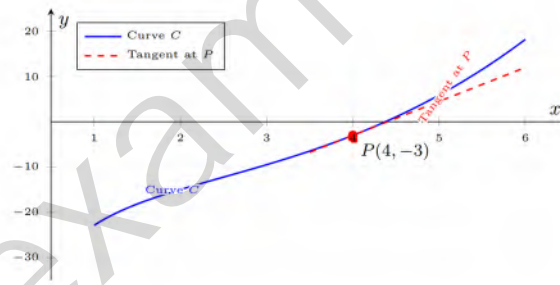
- **Calculate the gradient m at $x = 4$:** Substitute $x = 4$ into the derivative found in part (a):

$$\begin{aligned}
 m &= \frac{dy}{dx} \Big|_{x=4} \\
 &= \frac{3}{8}(4)^2 + 12(4)^{-3/2} \\
 &= \frac{3}{8}(16) + 12\left(\frac{1}{\sqrt{4^3}}\right) \\
 &= 6 + 12\left(\frac{1}{8}\right) \\
 &= 6 + 1.5 \\
 &= 7.5
 \end{aligned}$$

In fractional form, $m = \frac{15}{2}$.

- **Determine the equation of the line:** Using the **point-slope form** $y - y_1 = m(x - x_1)$ with point $P(4, -3)$:

$$\begin{aligned}
 y - (-3) &= 7.5(x - 4) \\
 y + 3 &= 7.5x - 30 \\
 y &= 7.5x - 33
 \end{aligned}$$



(b) $y = 7.5x - 33$

WMA11_P1(IAL)_Summer_2019_Q2

Solution

1. Rationalizing the Denominator

To simplify the expression $\frac{1}{4-2\sqrt{2}}$, we must **rationalize the denominator** by multiplying both the numerator and the denominator by the **conjugate** of the denominator, which is $4 + 2\sqrt{2}$.

- Multiply by the conjugate:

$$\begin{aligned}\frac{1}{4-2\sqrt{2}} &= \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}} \\ &= \frac{4+2\sqrt{2}}{(4)^2 - (2\sqrt{2})^2}\end{aligned}$$

- Simplify the denominator using the **difference of two squares** identity $(A - B)(A + B) = A^2 - B^2$:

$$\begin{aligned}(4)^2 - (2\sqrt{2})^2 &= 16 - (4 \times 2) \\ &= 16 - 8 \\ &= 8\end{aligned}$$

- Substitute back into the expression:

$$\begin{aligned}\frac{4+2\sqrt{2}}{8} &= \frac{4}{8} + \frac{2\sqrt{2}}{8} \\ &= \frac{1}{2} + \frac{1}{4}\sqrt{2}\end{aligned}$$

The simplified form is $\boxed{\frac{1}{2} + \frac{1}{4}\sqrt{2}}$.

2. Solving the Equation

We are given the equation:

$$4x = 2\sqrt{2}x + 20\sqrt{2}$$

- Rearrange the equation to group all terms containing x on the left-hand side:

$$4x - 2\sqrt{2}x = 20\sqrt{2}$$

- Factor out x :

$$x(4 - 2\sqrt{2}) = 20\sqrt{2}$$

- Solve for x by dividing both sides by $(4 - 2\sqrt{2})$:

$$x = \frac{20\sqrt{2}}{4 - 2\sqrt{2}}$$

- Use the result from part (a), where we found that $\frac{1}{4-2\sqrt{2}} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$:

$$\begin{aligned}x &= 20\sqrt{2} \times \left(\frac{1}{4-2\sqrt{2}}\right) \\ &= 20\sqrt{2} \left(\frac{1}{2} + \frac{1}{4}\sqrt{2}\right)\end{aligned}$$

- Distribute $20\sqrt{2}$ across the terms inside the parentheses:

$$\begin{aligned}x &= (20\sqrt{2} \times \frac{1}{2}) + (20\sqrt{2} \times \frac{1}{4}\sqrt{2}) \\ &= 10\sqrt{2} + (5 \times 2) \\ &= 10\sqrt{2} + 10 \\ &= 10 + 10\sqrt{2}\end{aligned}$$

The solution to the equation is $\boxed{10 + 10\sqrt{2}}$

WMA11_P1(IAL)_Summer_2019_Q3

Solution

1. Analysis of the Perimeter

The garden is an L-shaped polygon with all angles being right angles. To find the **perimeter** (P), we sum the lengths of all six edges. Based on the diagram:

- The total height on the left is $5x$.
- The total width at the bottom is $6x - 2$.
- The top horizontal edge is $2x - 1$.
- The upper vertical edge is $2x$.

For a rectilinear shape like this, the sum of the horizontal segments must equal the total width, and the sum of the vertical segments must equal the total height.

- The missing horizontal edge (middle) is $(6x - 2) - (2x - 1) = 4x - 1$.
- The missing vertical edge (right) is $5x - 2x = 3x$.

The perimeter is the sum of all sides:

$$\begin{aligned} P &= 5x + (2x - 1) + 2x + (4x - 1) + 3x + (6x - 2) \\ &= 22x - 4 \end{aligned}$$

Given that the perimeter is greater than 29 m:

$$\begin{aligned} 22x - 4 &> 29 \\ 22x &> 33 \\ x &> \frac{33}{22} \\ x &> 1.5 \text{ m} \end{aligned}$$

2. Formation and Solution of the Quadratic Inequality

The **area** (A) of the garden can be calculated by splitting the shape into two rectangles. We can split it vertically into a left rectangle and a right rectangle:

- Left rectangle: width = $2x - 1$, height = $5x$.
- Right rectangle: width = $(6x - 2) - (2x - 1) = 4x - 1$, height = $5x - 2x = 3x$.

$$\begin{aligned} A &= (2x - 1)(5x) + (4x - 1)(3x) \\ &= (10x^2 - 5x) + (12x^2 - 3x) \\ &= 22x^2 - 8x \end{aligned}$$

Given that the area is less than 72 m^2 :

$$\begin{aligned} 22x^2 - 8x &< 72 \\ 22x^2 - 8x - 72 &< 0 \end{aligned}$$

Dividing by 2:

$$11x^2 - 4x - 36 < 0$$

To solve the **quadratic inequality**, we first find the roots of $11x^2 - 4x - 36 = 0$ using the **quadratic formula**:

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(11)(-36)}}{2(11)} \\ &= \frac{4 \pm \sqrt{16 + 1584}}{22} \\ &= \frac{4 \pm \sqrt{1600}}{22} \\ &= \frac{4 \pm 40}{22} \end{aligned}$$

The roots are $x_1 = \frac{44}{22} = 2$ and $x_2 = \frac{-36}{22} = -\frac{18}{11} \approx -1.64$. Since the parabola opens upwards ($11 > 0$), the expression is negative between the roots:

$$-1.64 < x < 2$$

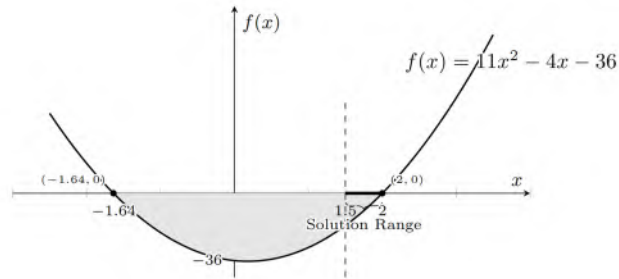
3. Range of Possible Values for x

We must satisfy all conditions simultaneously:

1. From the perimeter: $x > 1.5$
2. From the area: $x < 2$ (ignoring the negative root as x must be positive for physical lengths)
3. Physical constraints (all side lengths must be positive):
 - $2x - 1 > 0 \Rightarrow x > 0.5$
 - $6x - 2 > 0 \Rightarrow x > 1/3$
 - $4x - 1 > 0 \Rightarrow x > 0.25$

The most restrictive lower bound is $x > 1.5$. Combining this with the upper bound from the area inequality:

$$1.5 < x < 2$$



$$1.5 < x < 2$$

WMA11_P1(IAL)_Summer_2019_Q4

Solution

To find the indefinite integral of the given expression, we first simplify the integrand by distributing the denominator.

1. Simplify the integrand The integral is given by:

$$I = \int \frac{4x^2 + 1}{2\sqrt{x}} dx$$

We rewrite the square root as a power, $\sqrt{x} = x^{1/2}$, and divide each term in the numerator by the denominator:

$$\begin{aligned} \frac{4x^2 + 1}{2x^{1/2}} &= \frac{4x^2}{2x^{1/2}} + \frac{1}{2x^{1/2}} \\ &= 2x^{2-1/2} + \frac{1}{2}x^{-1/2} \\ &= 2x^{3/2} + \frac{1}{2}x^{-1/2} \end{aligned}$$

2. Integrate term by term Using the **Power Rule for Integration**, $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$):

$$\begin{aligned} I &= \int \left(2x^{3/2} + \frac{1}{2}x^{-1/2} \right) dx \\ &= 2 \left(\frac{x^{3/2+1}}{3/2+1} \right) + \frac{1}{2} \left(\frac{x^{-1/2+1}}{-1/2+1} \right) + C \\ &= 2 \left(\frac{x^{5/2}}{5/2} \right) + \frac{1}{2} \left(\frac{x^{1/2}}{1/2} \right) + C \end{aligned}$$

3. Simplify the coefficients Multiply by the reciprocals of the denominators:

$$\begin{aligned} I &= 2 \cdot \frac{2}{5}x^{5/2} + \frac{1}{2} \cdot 2x^{1/2} + C \\ &= \frac{4}{5}x^{5/2} + x^{1/2} + C \end{aligned}$$

The result can also be expressed in radical form:

$$I = \frac{4}{5}x^2\sqrt{x} + \sqrt{x} + C$$

Factoring out \sqrt{x} (or $x^{1/2}$):

$$I = \sqrt{x} \left(\frac{4}{5}x^2 + 1 \right) + C$$

$\frac{4}{5}x^{5/2} + x^{1/2} + C$

WMA11_P1(IAL)_Summer_2019_Q5

Solution

1. Solving the cubic equation

To find the real solutions of the equation $2x^3 + 3x^2 - 35x = 0$, we first factor out the common term x :

$$x(2x^2 + 3x - 35) = 0$$

- This gives the first solution:

$$x = 0$$

- Next, we solve the quadratic factor $2x^2 + 3x - 35 = 0$ using the **quadratic formula** or by **factoring**. To factor, we look for two numbers that multiply to $2 \times (-35) = -70$ and add to 3. These numbers are 10 and -7 :

$$2x^2 + 10x - 7x - 35 = 0$$

$$2x(x + 5) - 7(x + 5) = 0$$

$$(2x - 7)(x + 5) = 0$$

- Setting each factor to zero:

$$2x - 7 = 0 \implies x = \frac{7}{2} = 3.5$$

$$x + 5 = 0 \implies x = -5$$

The real solutions for part (a) are $x = -5, 0, 3.5$.

2. Solving the higher-order equation

The equation $2(y - 5)^6 + 3(y - 5)^4 - 35(y - 5)^2 = 0$ can be solved by using a **substitution**. Let $x = (y - 5)^2$. Substituting this into the equation yields:

$$2x^3 + 3x^2 - 35x = 0$$

This is the same equation solved in part (a). Therefore, the possible values for x are the solutions found previously:

$$x = -5, \quad x = 0, \quad x = 3.5$$

Since $x = (y - 5)^2$, and the square of a real number must be non-negative ($x \geq 0$), we discard $x = -5$. We solve for y using the remaining values:

- **Case 1:** $x = 0$

$$(y - 5)^2 = 0$$

$$y - 5 = 0$$

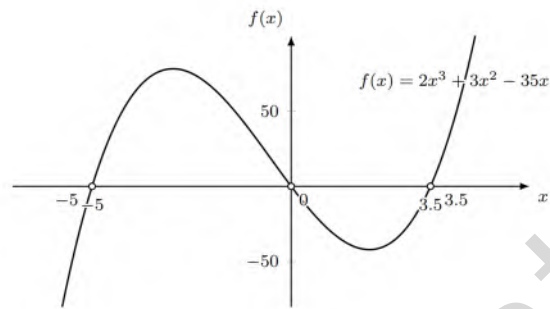
$$y = 5$$

- **Case 2:** $x = 3.5$

$$(y - 5)^2 = 3.5$$

$$y - 5 = \pm\sqrt{3.5}$$

$$y = 5 \pm \sqrt{3.5}$$



The real solutions for part (b) are $y = 5, 5 + \sqrt{3.5}, 5 - \sqrt{3.5}$.

Final Answers: (a) $x = -5, 0, 3.5$ (b) $y = 5, 5 \pm \sqrt{3.5}$

WMA11_P1(IAL)_Summer_2019_Q6

Solution

1. Determining the value of c

To find the value of the constant c such that the line $y = 4x + c$ meets the curve $y = x(x - 3)$ at exactly one point, we must equate the two expressions for y :

$$x(x - 3) = 4x + c$$

$$x^2 - 3x = 4x + c$$

$$x^2 - 7x - c = 0$$

This is a **quadratic equation** in the form $ax^2 + bx + d = 0$, where $a = 1$, $b = -7$, and $d = -c$. For the line to meet the curve at only one point, the line must be a **tangent** to the curve. This occurs when the **discriminant** (Δ) of the quadratic equation is equal to zero:

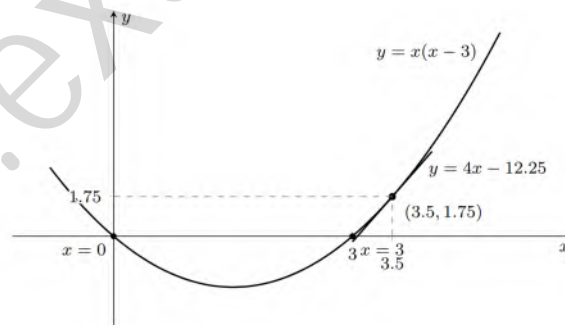
$$\Delta = b^2 - 4ad$$

$$0 = (-7)^2 - 4(1)(-c)$$

$$0 = 49 + 4c$$

$$-4c = 49$$

$$c = -12.25$$



(a) The value of c is:

$$c = -12.25$$

2. Finding the coordinates of the point of intersection

Substitute the value of $c = -12.25$ back into the quadratic equation derived in step 1:

$$x^2 - 7x - (-12.25) = 0$$

$$x^2 - 7x + 12.25 = 0$$

Since the discriminant is zero, this equation is a **perfect square trinomial**:

$$(x - 3.5)^2 = 0$$

$$x = 3.5$$

To find the corresponding y -coordinate, substitute $x = 3.5$ into the equation of the line:

$$y = 4(3.5) - 12.25$$

$$y = 14 - 12.25$$

$$y = 1.75$$

(b) The coordinates of the point of intersection are:

$$(3.5, 1.75)$$

WMA11_P1(IAL)_Summer_2019_Q7

Solution

1. Area of the sector $ABCOA$

The shape $ABCOA$ is a sector of a circle with center O and radius $r = 6$ cm. The total angle around point O is 2π radians. The triangle AOD is formed by the radius OA and the segment OD , which lies on the radius OC . The angle $\angle AOD$ is given as 0.7 rad. Therefore, the angle of the sector $ABCOA$, denoted as θ_{sector} , is the reflex angle:

$$\theta_{\text{sector}} = 2\pi - 0.7$$

The **area of a sector** is calculated using the formula $A = \frac{1}{2}r^2\theta$:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot 6^2 \cdot (2\pi - 0.7) \\ &= 18 \cdot (2\pi - 0.7) \\ &\approx 100.497 \end{aligned}$$

Rounding to one decimal place, we obtain:

$$\boxed{100.5 \text{ cm}^2}$$

2. Size of angle ADO

In triangle AOD , we have the following known values:

- Side $OA = r = 6$ cm
- Side $AD = 5$ cm
- Angle $\angle AOD = 0.7$ rad

We apply the **Sine Rule** to find the angle $\angle ADO$:

$$\begin{aligned} \frac{\sin(\angle ADO)}{OA} &= \frac{\sin(\angle AOD)}{AD} \\ \sin(\angle ADO) &= \frac{6 \cdot \sin(0.7)}{5} \\ \sin(\angle ADO) &\approx 0.77306 \end{aligned}$$

Calculating the arcsine gives the acute solution:

$$\angle ADO_{\text{acute}} = \arcsin(0.77306) \approx 0.883 \text{ rad}$$

Since the problem states that $\angle ADO$ is **obtuse**, we find the supplementary angle:

$$\begin{aligned} \angle ADO &= \pi - 0.88347... \\ &\approx 2.25812 \end{aligned}$$

Rounding to three decimal places:

$$\boxed{2.258 \text{ rad}}$$

3. Perimeter of shape $ABCD$

The perimeter consists of the arc ABC , the segment CD , and the segment DA .

- **Arc length ABC :**

$$\begin{aligned} s &= r \cdot \theta_{\text{sector}} \\ &= 6 \cdot (2\pi - 0.7) \\ &\approx 33.499 \text{ cm} \end{aligned}$$

- **Length of CD :** First, we find the length OD using the **Sine Rule** or the **Cosine Rule**. Using the sum of angles in triangle AOD :

$$\begin{aligned} \angle OAD &= \pi - 0.7 - 2.25812 \\ &= 0.18347 \text{ rad} \end{aligned}$$

Using the Sine Rule for OD :

$$\begin{aligned} \frac{OD}{\sin(0.18347)} &= \frac{5}{\sin(0.7)} \\ OD &= \frac{5 \cdot \sin(0.18347)}{\sin(0.7)} \\ &\approx 1.412 \text{ cm} \end{aligned}$$

Since D lies on OC and OC is a radius (6 cm):

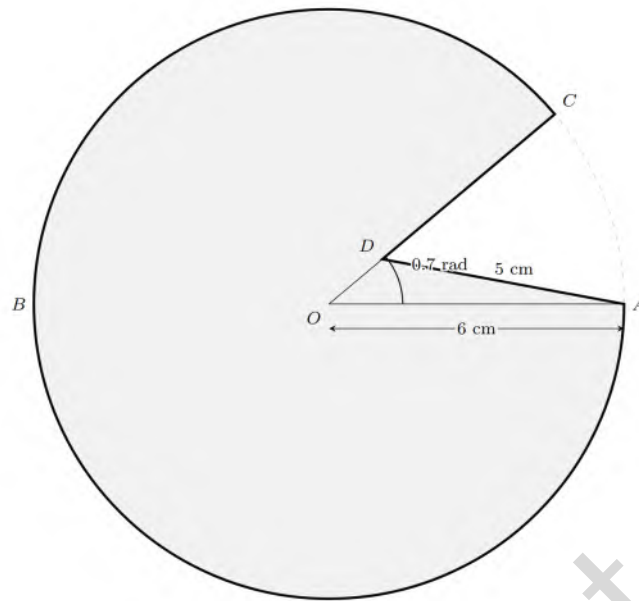
$$\begin{aligned} CD &= OC - OD \\ &= 6 - 1.412 \\ &= 4.588 \text{ cm} \end{aligned}$$

- **Total Perimeter:**

$$\begin{aligned} P &= \text{Arc } ABC + CD + DA \\ &= 33.499 + 4.588 + 5 \\ &= 43.087 \end{aligned}$$

Rounding to one decimal place:

$$\boxed{43.1 \text{ cm}}$$



WMA11_P1(IAL)_Summer_2019_Q8

Solution

1. Equation of the normal to C at P

- First, we determine the **gradient** of the **tangent** to the curve at point $P(4, 1)$. The gradient of the tangent is given by the value of the derivative $f'(x)$ at $x = 4$:

$$\begin{aligned} f'(x) &= 4\sqrt{x} - 2 - \frac{8}{3x^2} \\ f'(4) &= 4\sqrt{4} - 2 - \frac{8}{3(4)^2} \\ &= 4(2) - 2 - \frac{8}{3(16)} \\ &= 8 - 2 - \frac{1}{6} \\ &= 6 - \frac{1}{6} \\ &= \frac{35}{6} \end{aligned}$$

- The **normal** to a curve at a point is perpendicular to the tangent at that point. If the gradient of the tangent is m_t , the gradient of the normal m_n satisfies $m_t \cdot m_n = -1$.

$$\begin{aligned} m_n &= -\frac{1}{f'(4)} \\ &= -\frac{1}{35/6} \\ &= -\frac{6}{35} \end{aligned}$$

- Using the **point-slope form** of a linear equation, $y - y_1 = m(x - x_1)$, with point $P(4, 1)$ and gradient $m_n = -6/35$:

$$\begin{aligned} y - 1 &= -\frac{6}{35}(x - 4) \\ 35(y - 1) &= -6(x - 4) \\ 35y - 35 &= -6x + 24 \\ 6x + 35y - 59 &= 0 \end{aligned}$$

The equation of the normal is $\boxed{6x + 35y - 59 = 0}$.

2. Finding $f(x)$

- To find $f(x)$, we perform **indefinite integration** on the derivative $f'(x)$:

$$\begin{aligned}
 f(x) &= \int f'(x) dx \\
 &= \int \left(4x^{1/2} - 2 - \frac{8}{3}x^{-2} \right) dx \\
 &= 4 \left(\frac{x^{3/2}}{3/2} \right) - 2x - \frac{8}{3} \left(\frac{x^{-1}}{-1} \right) + C \\
 &= \frac{8}{3}x^{3/2} - 2x + \frac{8}{3x} + C
 \end{aligned}$$

- We use the given point $P(4, 1)$ to solve for the **constant of integration** C . Substituting $x = 4$ and $f(x) = 1$:

$$1 = \frac{8}{3}(4)^{3/2} - 2(4) + \frac{8}{3(4)} + C$$

$$1 = \frac{8}{3}(8) - 8 + \frac{2}{3} + C$$

$$1 = \frac{64}{3} - \frac{24}{3} + \frac{2}{3} + C$$

$$1 = \frac{42}{3} + C$$

$$1 = 14 + C$$

$$C = -13$$

- Substituting C back into the expression for $f(x)$:

$$f(x) = \frac{8}{3}x^{3/2} - 2x + \frac{8}{3x} - 13$$

The function is $f(x) = \frac{8}{3}x\sqrt{x} - 2x + \frac{8}{3x} - 13$

WMA11_P1(IAL)_Summer_2019_Q9

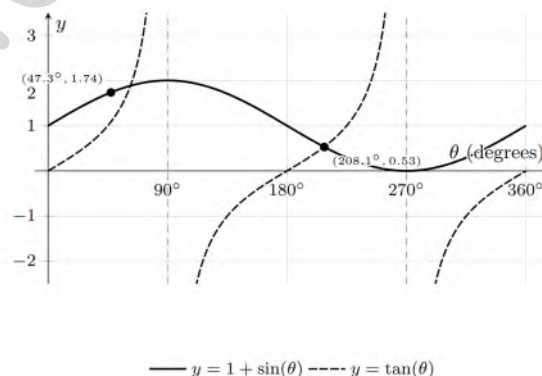
Solution

1. Analysis of the Curves

To determine the number of solutions for the equation $\tan \theta = 1 + \sin \theta$, we first examine the behavior of the functions $y_1 = 1 + \sin \theta$ and $y_2 = \tan \theta$ within one full period, $0^\circ \leq \theta \leq 360^\circ$.

- **Curve (i):** $y_1 = 1 + \sin \theta$ This is a **sine wave** shifted upward by 1 unit.
 - At $\theta = 0^\circ$, $y_1 = 1$.
 - At $\theta = 90^\circ$, $y_1 = 2$ (maximum).
 - At $\theta = 180^\circ$, $y_1 = 1$.
 - At $\theta = 270^\circ$, $y_1 = 0$ (minimum).
 - At $\theta = 360^\circ$, $y_1 = 1$.
- **Curve (ii):** $y_2 = \tan \theta$ This is the **tangent function**, which has vertical asymptotes at $\theta = 90^\circ$ and $\theta = 270^\circ$.
 - In the interval $[0^\circ, 90^\circ)$, y_2 starts at 0 and goes to $+\infty$. Since y_1 starts at 1 and goes to 2, there is exactly **one intersection** (at approximately $\theta \approx 48.2^\circ$).
 - In the interval $(90^\circ, 270^\circ)$, y_2 goes from $-\infty$ to $+\infty$. Since y_1 stays between 0 and 2, there is exactly **one intersection** (at approximately $\theta \approx 225.0^\circ$).
 - In the interval $(270^\circ, 360^\circ]$, y_2 goes from $-\infty$ to 0. Since y_1 goes from 0 to 1, there are **no intersections** in this specific quadrant because $y_2 \leq 0$ and $y_1 \geq 0$, meeting only at 270° if defined, but $\tan \theta$ is undefined there.

Thus, there are **2 solutions** in every 360° cycle.



2. Number of solutions for $0^\circ \leq \theta \leq 2160^\circ$

The functions $y = 1 + \sin \theta$ and $y = \tan \theta$ both have a **period** of 360° (note: while $\tan \theta$ has a period of 180° , the combined system repeats every 360° due to the sine term).

- Total range: 2160° .
- Number of full cycles: $\frac{2160^\circ}{360^\circ} = 6$ cycles.
- Solutions per cycle: 2.
- Total solutions: $6 \times 2 = 12$.

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3. Number of solutions for $0^\circ \leq \theta \leq 1980^\circ$

We break the range into full cycles and the remaining partial cycle:

- Number of full cycles: $\lfloor \frac{1980}{360} \rfloor = 5$ full cycles.
- Degrees covered by 5 cycles: $5 \times 360^\circ = 1800^\circ$.
- Solutions in 5 cycles: $5 \times 2 = 10$.
- Remaining interval: $1980^\circ - 1800^\circ = 180^\circ$ (from 0° to 180° in the 6th cycle).

In the interval $[0^\circ, 180^\circ]$ of a standard cycle:

- There is 1 solution in $[0^\circ, 90^\circ)$.
- There are no solutions in $[90^\circ, 180^\circ]$ because in this region $y_1 = 1 + \sin \theta \geq 1$ while $y_2 = \tan \theta \leq 0$.
- Total solutions: $10 + 1 = 11$.

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WMA11_P1(IAL)_Summer_2019_Q10

Solution

1. Coordinates of the point P

The curve is defined by the function $f(x) = (x - 4)(2x + 1)^2$. The x -intercepts occur where $f(x) = 0$:

$$(x - 4)(2x + 1)^2 = 0$$

This yields two solutions: $x = 4$ and $x = -1/2$.

- At $x = 4$, the factor $(x - 4)$ has a multiplicity of 1, meaning the curve crosses the x -axis. This corresponds to point Q .
- At $x = -1/2$, the factor $(2x + 1)^2$ has a multiplicity of 2, meaning the curve **touches** the x -axis (a local extremum). This corresponds to point P .

Since P lies on the x -axis, its y -coordinate is 0.

$$P(-0.5, 0)$$

2. Finding the derivative $f'(x)$

To find $f'(x)$, we apply the **product rule**, which states that for $y = u \cdot v$, $y' = u'v + uv'$. Let $u = (x - 4)$ and $v = (2x + 1)^2$.

- $u' = 1$
- To find v' , we use the **chain rule**: $v' = 2(2x + 1) \cdot 2 = 4(2x + 1)$.

Now, substitute these into the product rule:

$$\begin{aligned} f'(x) &= (1)(2x + 1)^2 + (x - 4)(4)(2x + 1) \\ &= (2x + 1)[(2x + 1) + 4(x - 4)] \\ &= (2x + 1)(2x + 1 + 4x - 16) \\ &= (2x + 1)(6x - 15) \end{aligned}$$

Expanding the expression:

$$\begin{aligned} f'(x) &= 12x^2 - 30x + 6x - 15 \\ &= 12x^2 - 24x - 15 \end{aligned}$$

$$f'(x) = 12x^2 - 24x - 15$$

3. Tangent equation at $x = 5/2$

To find the **tangent** equation, we first evaluate the gradient at $x = 2.5$ using $f'(x)$:

$$\begin{aligned}f'(2.5) &= 12(2.5)^2 - 24(2.5) - 15 \\ &= 12(6.25) - 60 - 15 \\ &= 75 - 60 - 15 \\ &= 0\end{aligned}$$

Since the gradient is 0, the tangent is a horizontal line of the form $y = k$. We find k by calculating the y -coordinate at $x = 2.5$:

$$\begin{aligned}f(2.5) &= (2.5 - 4)(2(2.5) + 1)^2 \\ &= (-1.5)(5 + 1)^2 \\ &= (-1.5)(6)^2 \\ &= -1.5 \cdot 36 \\ &= -54\end{aligned}$$

Thus, the equation of the tangent is $y = -54$.

$$\boxed{k = -54}$$

4. Possible values of a

The curve $y = f(x + a)$ represents a horizontal **translation** of the original curve $y = f(x)$ by a units to the left. If this new curve passes through the origin $(0, 0)$, then:

$$f(0 + a) = 0 \implies f(a) = 0$$

This means a must be an x -intercept of the original function $f(x)$. From step 1, the x -intercepts are $x = 4$ and $x = -0.5$. Therefore, the possible values for the constant a are:

$$\boxed{a = 4, a = -0.5}$$