

# PURE MATHEMATICS

UNIT P3(IAL)

2020 — 2025

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1 - (WMA11/P3(IAL)\_Summer\_2020\_Q3) - Algebra And Functions

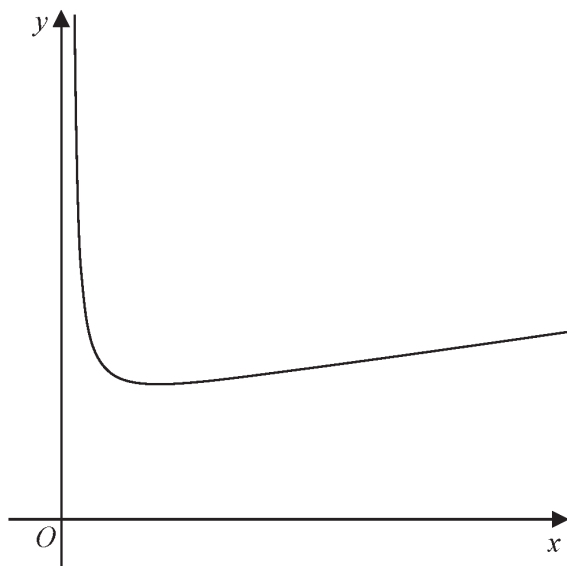


Figure 1

Figure 1 shows a sketch of a curve with equation  $y = f(x)$  where

$$f(x) = \frac{2x + 3}{\sqrt{4x - 1}} \quad x > \frac{1}{4}$$

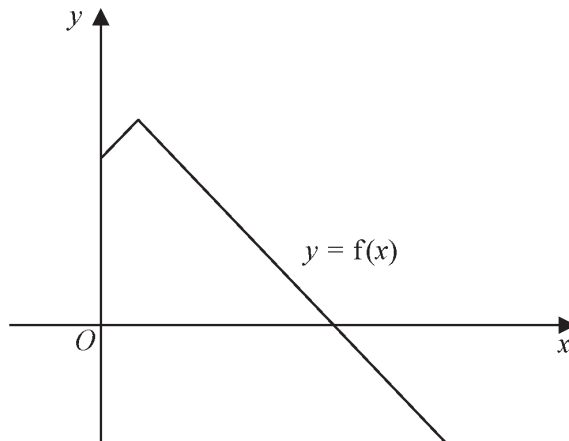
(a) Find, in simplest form,  $f'(x)$ .

(4)

(b) Hence find the range of  $f$ .

(3)

2 - (WMA11/P3(IAL)\_Summer\_2020\_Q4) - Algebra And Functions



**Figure 2**

Figure 2 shows a sketch of part of the graph with equation  $y = f(x)$  where

$$f(x) = 21 - 2|2 - x| \quad x \geq 0$$

(a) Find  $ff(6)$  (2)

(b) Solve the equation  $f(x) = 5x$  (2)

Given that the equation  $f(x) = k$ , where  $k$  is a constant, has exactly two roots,

(c) state the set of possible values of  $k$ . (2)

The graph with equation  $y = f(x)$  is transformed onto the graph with equation  $y = af(x - b)$

The vertex of the graph with equation  $y = af(x - b)$  is  $(6, 3)$ .

Given that  $a$  and  $b$  are constants,

(d) find the value of  $a$  and the value of  $b$ . (2)

**3** - (WMA11/P3(IAL)\_Winter\_2020\_Q1) - Algebra And Functions

A population of a rare species of toad is being studied.

The number of toads,  $N$ , in the population,  $t$  years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \quad t \geq 0, t \in \mathbb{R}$$

According to this model,

- (a) calculate the number of toads in the population at the start of the study, (1)
- (b) find the value of  $t$  when there are 420 toads in the population, giving your answer to 2 decimal places. (4)
- (c) Explain why, according to this model, the number of toads in the population can never reach 500 (1)

**4** - (WMA11/P3(IAL)\_Winter\_2020\_Q2) - Algebra And Functions

The function  $f$  and the function  $g$  are defined by

$$f(x) = \frac{12}{x+1} \quad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2} \ln x \quad x > 0, x \in \mathbb{R}$$

- (a) Find, in simplest form, the value of  $fg(e^2)$  (2)
- (b) Find  $f^{-1}$  (3)
- (c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x) \quad (3)$$

# ANSWERS

[www.exam-mate.com](http://www.exam-mate.com)

## 1 - (WMA11/P3(IAL)\_Summer\_2020\_Q3) - Algebra And Functions

(a)	$\frac{dy}{dx} = \frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$ $\frac{(4x-1)^{\frac{1}{2}} \times 2 - (2x+3) \times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)} \times \frac{(4x-1)^{\frac{1}{2}}}{(4x-1)^{\frac{1}{2}}} = \frac{4x-8}{(4x-1)^{\frac{3}{2}}}$	M1 A1  dM1 A1  (4)
(b)	Turning point where $\frac{dy}{dx} = 0 \Rightarrow x = 2$ Find value of f at $x = 2 \Rightarrow f(x) = \sqrt{7}$ Hence range is $f \geq \sqrt{7}$	M1  dM1 A1  (3) <b>(7 marks)</b>

## 2 - (WMA11/P3(IAL)\_Summer\_2020\_Q4) - Algebra And Functions

(a)	$ff(6) = f(13) = -1$	M1 A1  (2)
(b)	Attempts $21 + 2(2-x) = 5x \Rightarrow x = \dots$ or $21 - 2(x-2) = 5x \Rightarrow x = \dots$ $x = \frac{25}{7}$ only	M1  A1  (2)
(c)	Either $k < 21$ or $k \geq 17$ $17 \leq k < 21$	M1  A1  (2)
(d)	$a = \frac{1}{7}$ $b = 4$	B1 B1  (2) <b>(8 marks)</b>

## 3 - (WMA11/P3(IAL)\_Winter\_2020\_Q1) - Algebra And Functions

(a)	$P_0 = 300$	B1  (1)
(b)	$420 = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \Rightarrow 60e^{0.12t} = 420$ Correct use of lns $\Rightarrow t = \frac{\ln 7}{0.12} = 16.22$	M1 A1  dM1 A1  (4)
(c)	States that maximum number (upper limit) is 450 so cannot reach 500	B1  (1) <b>6 marks</b>

## 4 - (WMA11/P3(IAL)\_Winter\_2020\_Q2) - Algebra And Functions

(a)	$fg(e^2) = f\left(\frac{5}{2}\ln e^2\right) = \frac{12}{\frac{5}{2}\ln e^2 + 1} = 2$	M1, A1 (2)
(b)	$f(x) = \frac{12}{x+1}$ $f^{-1}(x) = \frac{12}{x} - 1$ $0 < x < 12$	M1 A1 B1 (3)
(c)	$\frac{12}{x+1} = \frac{12}{x} - 1 \Rightarrow 12x = 12(x+1) - x(x+1)$ $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ <p style="text-align: right;">Must be 3TQ</p> $x = 3 \text{ only}$	M1 dM1 A1 (3) <b>8 marks</b>
(c) Alts	<p>Solves <math>f^{-1}(x) = x \Rightarrow \frac{12}{x} - 1 = x</math> leading to quadratic equation,</p> <p>or solves <math>f(x) = x \Rightarrow \frac{12}{x+1} = x</math> leading to quadratic equation</p> $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = \dots$ <p style="text-align: right;">Must be 3TQ</p> $x = 3 \text{ only}$	M1 dM1 A1 (3)

## 5 - (WMA11/P3(IAL)\_Winter\_2020\_Q4) - Algebra And Functions, Trigonometry

(i) (a)	$f'(x) = \frac{4(x-3)(2x+5) - (2x+5)^2}{(x-3)^2} \text{ or } \frac{(x-3)(8x+20) - (4x^2 + 20x + 25)}{(x-3)^2}$ $= \frac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1 M1 A1
(b)	<p>Attempts both critical values or finds one "correct" end</p> $x < -2.5, x > 8.5 \text{ (accept } x \leq -2.5, x \geq 8.5)$	M1 A1 (6)
(ii)	<p>Attempts the chain rule on <math>(\sin 4x)^{\frac{1}{2}} \rightarrow A(\sin 4x)^{\frac{1}{2}} \times \cos 4x</math></p> $g(x) = x(\sin 4x)^{\frac{1}{2}} \Rightarrow g'(x) = (\sin 4x)^{\frac{1}{2}} + x \times \frac{1}{2}(\sin 4x)^{-\frac{1}{2}} 4 \cos 4x$ <p>Sets <math>g'(x) = 0 \rightarrow (\sin 4x)^{\frac{1}{2}} + x \times \frac{2 \cos 4x}{(\sin 4x)^{\frac{1}{2}}} = 0</math> and <math>\times \frac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}</math> oe</p> $\rightarrow \tan 4x + 2x = 0$	M1 M1 A1 M1 A1 (5) <b>11 marks</b>