

PURE MATHEMATICS

UNIT P2(IAL)

2020 — 2025

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1 - (WMA11/P2(IAL)_Summer_2020_Q3) - Algebra And Functions, Differentiation

$$f(x) = ax^3 - x^2 + bx + 4$$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b .

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

2 - (WMA11/P2(IAL)_Winter_2020_Q3) - Algebra And Functions, Trigonometry

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6 \tan^3 \theta + 17 \tan^2 \theta + 4 \tan \theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

3 - (WMA11/P2(IAL)_Winter_2021_Q1) - Algebra And Functions

$$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b .

(4)

4 - (WMA11/P2(IAL)_Summer_2022_Q7) - Algebra And Functions, Integration

$$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

• $(x + 2)$ is a factor of $f(x)$

• $\int_3^5 f(x) dx = 176$

find the value of A and the value of B .

(7)

ANSWERS

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1 - (WMA11/P2(IAL)_Summer_2020_Q3) - Algebra And Functions, Differentiation

(a)	$a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$ <p>Attempts to set $f(-4) = -108$ to obtain an equation in a and b. Score when you see “- 4” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $-64a - 16 - 4b + 4 = -108$ Condone minor slips on the lhs e.g. one sign error between terms but must use - 108</p>	M1		
	<p>As an alternative for the first mark we will condone an attempt at long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(x + 4)$ to obtain a remainder in terms of a and b which is then equated to -108 For reference, the quotient is $ax^2 - (1+4a)x + 16a + b + 4$ and the remainder is $-4b - 64a - 12$</p>			
	$-64a - 16 - 4b + 4 = -108$ $\Rightarrow 16a + b = 24^*$	<p>Correct equation obtained with no errors and at least one line of intermediate working if starting with e.g. $a(-4)^3 - (-4)^2 + b(-4) + 4 = -108$</p>	A1*	
(2)				
(b)	$a\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 4 = 0$ <p>Attempts to set $f\left(\frac{1}{2}\right) = 0$ to obtain an equation in a and b. Condone slips. Score when you see “$\frac{1}{2}$” embedded in the equation or 2 correct terms (excluding the “+ 4”) on lhs. May be implied by e.g. $\frac{a}{8} - \frac{1}{4} + \frac{b}{2} + 4 = 0$ The “= 0” may be implied when they attempt to solve simultaneously below</p>	M1		
	<p>An alternative for the first mark is to attempt long division. This requires a complete method to divide $(ax^3 - x^2 + bx + 4)$ by $(2x - 1)$ to obtain a remainder in a and b which is then equated to 0 For reference, the quotient is $\frac{a}{2}x^2 + \left(\frac{a-1}{4} - \frac{1}{2}\right)x + \left(\frac{b-1}{2} - \frac{1}{4} + \frac{a}{8}\right)$ and the remainder is $\frac{15}{4} + \frac{b}{2} + \frac{a}{8}$</p>			
	$16a + b = 24, a + 4b = -30$ $\Rightarrow a = \dots, b = \dots$	<p>Attempts to solve $16a + b = 24$ simultaneously with their equation in a and b. This may be implied if values of a and b are obtained (e.g. calculator)</p>	M1	
	$a = 2, b = -8$	<p>Correct values</p>	A1	
(3)				
(c)	$f(x) = 2x^3 - x^2 - 8x + 4$ $\Rightarrow f'(x) = 6x^2 - 2x - 8$	<p>Correct derivative (follow through their a and b). Allow unsimplified and apply isw if necessary. Allow with the letters “a” and “b” and a “made up” “a” and “b”.</p>	B1ft	
(1)				

(d)	$6x^2 - 2x - 8 = 0$ $\Rightarrow (3x - 4)(x + 1) = 0$ $\Rightarrow x = \dots$	Sets their $f'(x) = 0$ (may be implied) and solves a 3 term quadratic. Apply general guidance if necessary. You may need to check if a calculator has been used.	M1
	$x = \frac{4}{3}, -1 \Rightarrow y = \dots$	Uses at least one of their x values to find a value for y using their $f(x)$ <u>where x is from an attempt to solve $f'(x) = 0$</u> . You may need to check their y values if working is not shown.	M1
	$\left(\frac{4}{3}, -\frac{100}{27}\right)$ or $(-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$		A1
	One correct point. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		
	$\left(\frac{4}{3}, -\frac{100}{27}\right)$ and $(-1, 9)$ Or e.g. $x = \frac{4}{3}, y = -\frac{100}{27}$ and $x = -1, y = 9$		A1
	Both correct points. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. Depends on having scored both previous M marks.		
	Fully correct answers with no working scores 4/4 following a <u>correct</u> part (c) i.e. $\Rightarrow f'(x) = 6x^2 - 2x - 8$		
			(4)
			Total 10

2 - (WMA11/P2(IAL)_Winter_2020_Q3) - Algebra And Functions, Trigonometry

(a)	Attempts $f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 17\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) - 12$ $= 0 \Rightarrow (2x + 3)$ is a factor *	M1 A1*	(2)
(b)	$6x^3 + 17x^2 + 4x - 12 = (2x + 3)(3x^2 + 4x - 4)$ $= (2x + 3)(3x - 2)(x + 2)$	M1 A1 dM1 A1	(4)
(c)	Solves $\tan \theta = -\frac{3}{2}$ or “-2” or “ $\frac{2}{3}$ ” $\theta = \text{awrt } 2.03, 2.16$	M1 A1	(2)
			(8 marks)

3 - (WMA11/P2(IAL)_Winter_2021_Q1) - Algebra And Functions

(a)	$f(-1) = (-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) + 5 = 4$	M1
	$1 - a - 3 - b + 5 = 4 \Rightarrow a + b = -1$ *	A1*
		(2)
(b)	$f(2) = (2)^4 + a(2)^3 - 3(2)^2 + b(2) + 5 = -23$	M1
	$\Rightarrow 8a + 2b = -32$ oe (eg $4a + b = -16$)	A1
	$b = -1 - a \Rightarrow 4a - 1 - a = -16 \Rightarrow a = \dots$	dM1
	$a = -5, b = 4$	A1
		(4)
	(6 marks)	

4 - (WMA11/P2(IAL)_Summer_2022_Q7) - Algebra And Functions, Integration

	$(x + 2)$ a factor $\Rightarrow f(-2) = 0 \Rightarrow -8A + 24 + 8 + B = 0$	M1A1
	$\int f(x) dx = \frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx$	M1 A1
	$\int_3^5 f(x) dx = 176 \Rightarrow \left[\frac{A}{4}x^4 + 2x^3 - 2x^2 + Bx \right]_3^5 = 176$ $\Rightarrow \left(\frac{A}{4}5^4 + 2(5^3) - 2(5^2) + 5B \right) - \left(\frac{A}{4}3^4 + 2(3^3) - 2(3^2) + 3B \right) = 176$	dM1
	$\left. \begin{array}{l} 8A - B = 32 \\ 136A + 2B = 12 \end{array} \right\} \Rightarrow A = \dots, B = \dots$	dM1
	$A = \frac{1}{2}, B = -28$	A1
		(7)
		(7 marks)