

# PURE MATHEMATICS

UNIT P1(IAL)

2020 — 2025

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1 - (WMA11/P1(IAL)\_Winter\_2020\_Q5) - Algebra And Functions

(a) Find, using algebra, all solutions of

$$20x^3 - 50x^2 - 30x = 0 \quad (3)$$

(b) Hence find all real solutions of

$$20(y+3)^{\frac{3}{2}} - 50(y+3) - 30(y+3)^{\frac{1}{2}} = 0 \quad (4)$$

2 - (WMA11/P1(IAL)\_Winter\_2020\_Q8) - Algebra And Functions

The straight line  $l$  has equation  $y = k(2x - 1)$ , where  $k$  is a constant.

The curve  $C$  has equation  $y = x^2 + 2x + 11$

Find the set of values of  $k$  for which  $l$  does not cross or touch  $C$ .

(6)

3 - (WMA11/P1(IAL)\_Winter\_2020\_Q10) - Algebra And Functions

The curve  $C_1$  has equation  $y = f(x)$ , where

$$f(x) = (4x - 3)(x - 5)^2$$

(a) Sketch  $C_1$  showing the coordinates of any point where the curve touches or crosses the coordinate axes.

(3)

(b) Hence or otherwise

(i) find the values of  $x$  for which  $f\left(\frac{1}{4}x\right) = 0$

(ii) find the value of the constant  $p$  such that the curve with equation  $y = f(x) + p$  passes through the origin.

(2)

A second curve  $C_2$  has equation  $y = g(x)$ , where  $g(x) = f(x + 1)$

(c) (i) Find, in simplest form,  $g(x)$ . You may leave your answer in a factorised form.

(ii) Hence, or otherwise, find the  $y$  intercept of curve  $C_2$

(3)

4 - (WMA11/P1(IAL)\_Summer\_2021\_Q5) - Algebra And Functions

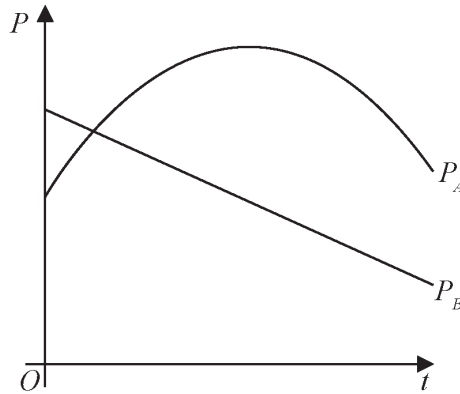


Figure 2

The share value of two companies, company *A* and company *B*, has been monitored over a 15-year period.

The share value  $P_A$  of **company A**, in millions of pounds, is modelled by the equation

$$P_A = 53 - 0.4(t - 8)^2 \quad t \geq 0$$

where  $t$  is the number of years after monitoring began.

The share value  $P_B$  of **company B**, in millions of pounds, is modelled by the equation

$$P_B = -1.6t + 44.2 \quad t \geq 0$$

where  $t$  is the number of years after monitoring began.

Figure 2 shows a graph of both models.

**Use the equations of one or both models to answer parts (a) to (d).**

- (a) Find the difference between the share value of **company A** and the share value of **company B** at the point monitoring began. (2)
- (b) State the maximum share value of **company A** during the 15-year period. (1)
- (c) Find, using algebra and showing your working, the times during this 15-year period when the share value of **company A** was greater than the share value of **company B**. (4)
- (d) Explain why the model for **company A** should not be used to predict its share value when  $t = 20$  (1)

5 - (WMA11/P1(IAL)\_Summer\_2021\_Q9) - Algebra And Functions

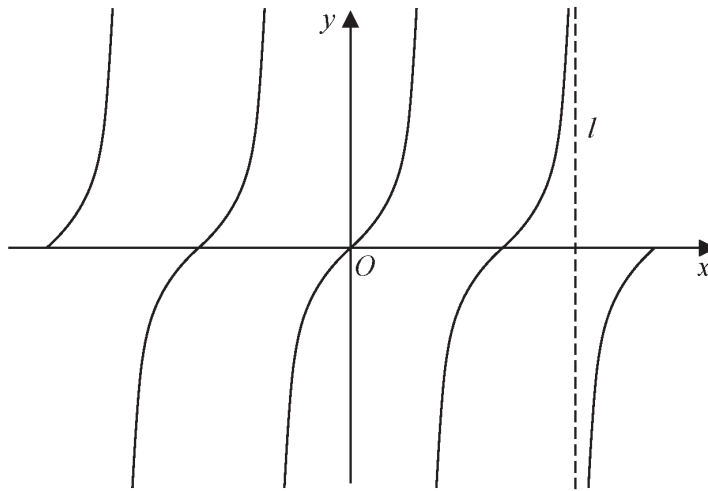


Figure 4

Figure 4 shows a sketch of the curve with equation

$$y = \tan x \quad -2\pi \leq x \leq 2\pi$$

The line  $l$ , shown in Figure 4, is an asymptote to  $y = \tan x$

(a) State an equation for  $l$ .

(1)

A copy of Figure 4, labelled Diagram 1, is shown on the next page.

(b) (i) On Diagram 1, sketch the curve with equation

$$y = \frac{1}{x} + 1 \quad -2\pi \leq x \leq 2\pi$$

stating the equation of the horizontal asymptote of this curve.

(ii) Hence, **giving a reason**, state the number of solutions of the equation

$$\tan x = \frac{1}{x} + 1$$

in the region  $-2\pi \leq x \leq 2\pi$

(4)

(c) State the number of solutions of the equation  $\tan x = \frac{1}{x} + 1$  in the region

(i)  $0 \leq x \leq 40\pi$

(ii)  $-10\pi \leq x \leq \frac{5}{2}\pi$

(2)

# ANSWERS

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## 1 - (WMA11/P1(IAL)\_Winter\_2020\_Q5) - Algebra And Functions

<b>(a)</b>	$20x^3 - 50x^2 - 30x = 0 \Rightarrow 10x(2x^2 - 5x - 3) = 0$ $\Rightarrow 10x(2x+1)(x-3) = 0$ $\Rightarrow x = 0, -\frac{1}{2}, 3$	M1 A1, A1  <b>(3)</b>
<b>(b)</b>	<p>Sets or implies <math>(y+3)^{\frac{1}{2}} = 0</math> or <math>-\frac{1}{2}</math> or 3</p> <p>Full method to find y</p> $y = 6$ $y = -3, 6$	B1ft  M1 A1ft, A1  <b>(4)</b> <b>(7 marks)</b>

## 2 - (WMA11/P1(IAL)\_Winter\_2020\_Q8) - Algebra And Functions

<p>Equates <math>y = k(2x-1)</math> and <math>y = x^2 + 2x + 11 \Rightarrow k(2x-1) = x^2 + 2x + 11</math></p> $\Rightarrow x^2 + (2-2k)x + 11+k (=0)$ <p>Attempts "<math>b^2 - 4ac</math>"...0 <math>\Rightarrow (2-2k)^2 - 4(11+k) \dots 0</math></p> <p>and proceeds to critical values</p> <p>Critical values of <math>(k =) 5, -2</math></p> <p>No roots so <math>b^2 - 4ac &lt; 0</math> so choose inside region</p> $-2 < k < 5$	M1 A1  M1 A1 M1 A1  <b>(6)</b> <b>(6 marks)</b>
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3 - (WMA11/P1(IAL)\_Winter\_2020\_Q10) - Algebra And Functions

<p><b>(a)</b></p>		<p>Shape for +ve <math>x^3</math></p> <p>Cuts <math>x</math>-axis at <math>(\frac{3}{4}, 0)</math> and meets at <math>(5, 0)</math></p> <p>Crosses <math>y</math>-axis at <math>(0, -75)</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p><b>(3)</b></p>
<p><b>(b)(i)</b></p>	<p><math>(x =) 3, 20</math></p>		<p>B1ft</p>
<p><b>(ii)</b></p>	<p><math>(p =) 75</math></p>		<p>B1ft</p>
<p><b>(c) (i)</b></p>	<p><math>(g(x) =) (4(x+1)-3)(x+1-5)^2 = (4x+1)(x-4)^2</math></p>		<p>M1 A1</p> <p><b>(2)</b></p>
<p><b>(ii)</b></p>	<p>16</p>		<p>B1</p> <p><b>(3)</b></p> <p><b>(8 marks)</b></p>

## 4 - (WMA11/P1(IAL)\_Summer\_2021\_Q5) - Algebra And Functions

(a)	$P_B - P_A = 44.2 - (53 - 0.4 \times 8^2) = \dots$ <p style="text-align: center;">awrt (£) 16.8 million</p>	M1 A1
		(2)
(b)	(£) 53 (million)	B1
		(1)
(c)	$-1.6t + 44.2 = 53 - 0.4(t - 8)^2$ $\Rightarrow 0.4t^2 - 8t + \frac{84}{5} = 0 \Rightarrow t = \dots$ $t = 10 - \sqrt{58} = \text{awrt } 2.38 \text{ (years)}$ <p style="text-align: center;">"2.38" &lt; t (≤ 15)</p>	M1 M1 A1
		A1ft
		(4)
	(d)	<p style="text-align: center;">"The <b>share value</b> would be negative" / "the <b>model</b> is known to hold for 15 years only (and 20 years is more than 15)."</p>
		(1)
		<b>(8 marks)</b>