IGCSE (9-1) Edexcel Past Papers

FURTHER PURE MATHEMATICS

Paper 2, 2R

2020 - 2025

Chapter 1	**LOGARITHMIC FUNCTIONS AND INDICES**	Page 1
Chapter 2	**THE QUADRATIC FUNCTION**	Page 5
Chapter 3	**IDENTITIES AND INEQUALITIES**	Page 13
Chapter 4	**GRAPHS**	Page 21
Chapter 5	**SERIES**	Page 46
Chapter 6	**THE BINOMIAL SERIES**	Page 53
Chapter 7	**SCALAR AND VECTOR QUANTITIES**	Page 56
Chapter 8	**RECTANGULAR CARTESIAN COORDINATES**	Page 67
Chapter 9	**CALCULUS**	Page 85
Chapter 10	**TRIGONOMETRY**	Page 129
Chapter 11	**RADIANS**	Page 151
	ANSWERS	Page 161

1 - (4PM1/2_Summer_2020_Q4) - *Logarithmic Functions And Indices*

(i) Solve the equation $16\log_r 4 = \log_4 r$

(2)

(ii) Solve the equation $\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = 1 + \log_5 x + \log_5 x^2$

(5)

2 - (4PM1/2_Summer_2020_Q8) - *Logarithmic Functions And Indices*

The curve C_1 has equation $y = 5e^{-2x} + 4$

The curve C_2 has equation $y = e^{2x}$

The curves C_1 and C_2 intersect at the point A.

(a) Find the exact coordinates of A.

(4)

The tangent at A to C_1 intersects the x-axis at the point B.

(b) Show that the x coordinate of B is $\frac{1}{2}(5 + \ln 5)$

(5)

The tangent at A to C_2 intersects the x-axis at the point D.

(c) Find the area of ΔABD.

(6)

3 - (4PM1/2R_Summer_2020_Q8) - *Logarithmic Functions And Indices*

Solve the equation $\log_3 x - 2\log_x 3 = 1$

(7)

4 - (4PM1/2_Summer_2021_Q8) - *Logarithmic Functions And Indices*

Use an algebraic method to solve the simultaneous equations

$$\log_4 a + 3\log_8 b = \frac{5}{2}$$

$$2^a = \frac{16^4}{4^{b^2}}$$

(8)

5 - (4PM1/2_Summer_2022_Q10) - *Logarithmic Functions And Indices*

Solve the equation

$$\log_4 x + \log_{16} x + \log_2 x = 10.5$$

Show your working clearly.

(5)

6 - (4PM1/2_Summer_2023_Q1) - *Logarithmic Functions And Indices*

Given that $\dfrac{a + 2\sqrt{5}}{3 - \sqrt{5}} = \dfrac{11 + b\sqrt{5}}{2}$ where a is an integer and b is prime,

find the value of a and the value of b
Show your working clearly.

(5)

7 - (4PM1/2R_Summer_2023_Q10) - *Logarithmic Functions And Indices*

Solve the equation

$$\log_4 x^3 + 8\log_x 64 = 22$$

(7)

8 - (4PM1/2R_Winter_2023_Q10) - *Logarithmic Functions And Indices*

(a) Show that $\dfrac{9^{3y}}{243} = 3^{(6y-5)}$

(4)

(b) Solve the simultaneous equations

$$\frac{9^{3y}}{243} = 27^{(x-2)}$$

$$\log_{10}\sqrt{6xy} = \log_4 2$$

(9)

9 - (4PM1/2_Winter_2024_Q6) - *Logarithmic Functions And Indices*

(i) Solve the equation $5(\log_b 9 + \log_b 3) = 3$

(4)

(ii) Solve the equation $3\log_3 x + 3\log_x 27 = 8\log_4 128$
Give your answers in exact form.

(7)

10 - (4PM1/2_Summer_2025_Q3) - *Logarithmic Functions And Indices*

Given that $\dfrac{a + b\sqrt{5}}{6 - 2\sqrt{5}} = \dfrac{9 + 4\sqrt{5}}{c}$ where a, b and c are prime numbers,

find the value of a, the value of b and the value of c

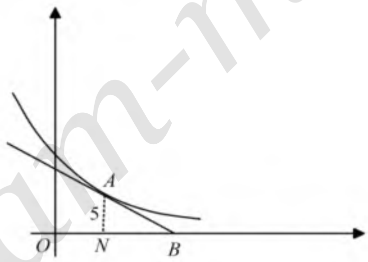
(5)

ANSWERS

1 - (4PM1/2_Summer_2020_Q4) - *Logarithmic Functions And Indices*

(i)	$\frac{16}{\log_4 r} = \log_4 r \Rightarrow 16 = (\log_4 r)^2 \Rightarrow \log_4 r = \pm 4$	M1
	$r = 4^4 = 256$ or $r = 4^{-4} = \frac{1}{256}$	A1 (2)
(ii)	$\log_5 9 + \log_5 12 + \log_5 15 + \log_5 18 = \log_5 (9 \times 12 \times 15 \times 18) = \log_5 29\,160$	M1
	$1 + \log_5 x + \log_5 x^2 = \log_5 5 + \log_5 x + \log_5 x^2 = \log_5 5x^3$	M1A1
	$5x^3 = 29\,160$	dM1
	$x = 18$	A1 (5) [7]
ALT 1	$\text{LHS} = \log_5 29\,160$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\left(\frac{\log_{10} 29\,160}{\log_{10} 5}\right) = 6.387...\left(= \log_5 x^3 + 1\right)$	A1
	$5.387... = 3\log_5 x$	dM1
	$\log_5 x = 1.795...$	
	$x = 18$	A1
ALT 2	$\text{LHS} = \log_5 29\,160$	M1
	$\text{RHS} = \log_5 5 + \log_5 x^3$	M1A1
	$\log_5 29\,160 = \log_5 5 + \log_5 5832$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 3	$\text{LHS} = \log_5 5832 + \log_5 5$	M1
	$\text{RHS} = 1 + \log_5 x^3$	M1
	$\text{LHS} = \log_5 5832 + 1$	A1
	$\log_5 5832 = \log_5 x^3$	
	$5832 = x^3$	dM1
	$x = 18$	A1
ALT 4	$\log_5 29\,160 - \log_5 x^3 = 1$	M1M1
	$\log_5 \frac{29\,160}{x^3} = 1$	A1
	$\frac{29\,160}{x^3} = 5 \Rightarrow x^3 = 5832$	dM1
	$x = 18$	A1

2 - (4PM1/2_Summer_2020_Q8) - *Logarithmic Functions And Indices*

(a)	$5e^{-2x}+4=e^{2x}\quad 5e^{-2x}+4-e^{2x}=0$ OR $y=\frac{5}{y}+4\Rightarrow y^2-4y-5=0$	M1	
	$\left(5e^{-x}-e^{x}\right)\left(e^{-x}+e^{x}\right)=0$ OR $(y-5)(y+1)=0$	M1	
	$5e^{-x}=e^{x}\quad e^{2x}=5\quad x=\frac{1}{2}\ln 5$ (oe eg $\ln\sqrt{5}$) OR $y=5$	A1	
	$\left(e^{-x}=-e^{x}\right.$ not possible$\left.\right)$ OR $e^{2x}=5\quad x=\frac{1}{2}\ln 5$		
	A is $\left(\frac{1}{2}\ln 5, 5\right)$	A1	(4)
(b)	$y=5e^{-2x}+4\Rightarrow\frac{dy}{dx}=-10e^{-2x}$	M1	
	At $A\quad\frac{dy}{dx}=-10e^{-2x}=-10\times\frac{1}{5}=-2$	A1ft	
	Eqn tgt: $y-5=-2\left(x-\frac{1}{2}\ln 5\right)$	dM1A1	
	$y=0\Rightarrow x=\frac{1}{2}(5+\ln 5)$ (= x coordinate of B)*	A1cso	(5)
ALT	For last 3 marks:		
	Hence $\frac{5}{NB}=2\Rightarrow NB=\frac{5}{2}$	dM1A1	
	$ON=\frac{1}{2}\ln 5$		
	$OB=\frac{1}{2}\ln 5+\frac{5}{2}=\frac{1}{2}\ \ 5+\ln 5$ * 	A1cso	
(c)	$C_2:\frac{dy}{dx}=2e^{2x}\Rightarrow$ grad tgt at A is $2\times 5=10$	B1ft	
	Eqn tgt: $y-5=10\left(x-\frac{1}{2}\ln 5\right)$	M1	
	At D: $x=\frac{1}{2}(-1+\ln 5)$	A1	
	Area $\Delta ABD=\frac{1}{2}\left(\frac{1}{2}(5+\ln 5)-\frac{1}{2}(-1+\ln 5)\right)\times 5$	M1A1	
	$=\frac{15}{2}$ or $7\frac{1}{2}$ (units^2)	A1	(6)
	See notes for area by "determinant" method		

ALT	For second and third marks:		
	$\frac{5}{ND}=10\Rightarrow ND=\frac{1}{2}$	M1	
	$OD=\frac{1}{2}\ln 5-\frac{1}{2}$	A1	[15]

3 - (4PM1/2R_Summer_2020_Q8) - *Logarithmic Functions And Indices*

	$\log_x 3 = \frac{1}{\log_3 x}$	B1
	Let $y = \log_3 x$ So $y - \frac{2}{y} = 1$	M1
	$y^2 - y - 2 = 0$	A1
	$(y-2)(y+1) = 0$	M1
	$\log_3 x = 2$ or $\log_3 x = -1$	M1
	$x = 9$ or $x = \frac{1}{3}$	A1 A1
		[7]

4 - (4PM1/2_Summer_2021_Q8) - *Logarithmic Functions And Indices*

	$\log_4 a + 2\log_4 b = \frac{5}{2}$	M1
	$\log_4 (ab^2) = \frac{5}{2}$	M1
	$32 = ab^2$	A1
	$2^a = \frac{2^{16}}{2^{2b^2}}$	M1
	$a = 16 - 2b^2$ or $b^2 = 8 - \frac{1}{2}a$	A1
	$32 = a(8 - \frac{1}{2}a)$ or $32 = (16 - 2b^2)b^2$	M1
	$a^2 - 16a + 64 = 0$ or $2b^4 - 16b^2 + 32 = 0$	A1
	$a = 8$ $b = 2$	A1
		Total 8 marks

5 - (4PM1/2_Summer_2022_Q10) - *Logarithmic Functions And Indices*

	$\frac{\log_2 x}{\log_2 4}+\frac{\log_2 x}{\log_2 16}+\log_2 x=10.5$	M1
	$\frac{\log_2 x}{2}+\frac{\log_2 x}{4}+\log_2 x=10.5$	M1
	$\frac{7}{4}\log_2 x=10.5$	M1
	$x=2^{'6'}$	M1
	$x=64$	A1 (5)
		Total 5 marks

6 - (4PM1/2_Summer_2023_Q1) - *Logarithmic Functions And Indices*

	$\frac{(a+2\sqrt{5})}{(3-\sqrt{5})}\times\frac{(3+\sqrt{5})}{(3+\sqrt{5})}=\frac{3a+a\sqrt{5}+6\sqrt{5}+10}{9-5}\left(=\frac{3a+10+(6+a)\sqrt{5}}{4}\right)$ $\left(\frac{3a+10+(6+a)\sqrt{5}}{4}=\frac{11+b\sqrt{5}}{2}\right)$ $\Rightarrow\frac{3a+10}{4}=\frac{11}{2}$ oe $\Rightarrow a=4\Rightarrow\frac{6+a}{4}=\frac{b}{2}$ oe $\Rightarrow b=5$	M1 M1M1A1A1 [5]
	ALT $\left(\frac{(a+2\sqrt{5})}{(3-\sqrt{5})}=\frac{11+b\sqrt{5}}{2}\Rightarrow\right)2(a+2\sqrt{5})=(3-\sqrt{5})(11+b\sqrt{5})$ $2a+4\sqrt{5}=33+3b\sqrt{5}-11\sqrt{5}-5b\left(=(33-5b)+(3b-11)\sqrt{5}\right)$ $\Rightarrow 4=3b-11\Rightarrow b=5$ $\Rightarrow 2a=33-5b\Rightarrow a=4$	M1 M1M1A1A1 [5]
		Total 5 marks

7 - (4PM1/2R_Summer_2023_Q10) - *Logarithmic Functions And Indices*

	$8\log_x 64 = \frac{8\log_4 64}{\log_4 x}$	M1
	$\log_4 x^3 = 3\log_4 x$	M1
	$\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \frac{8\log_4 64}{\log_4 x} = 22$	
	$\Rightarrow 3(\log_4 x)^2 + 8\log_4 64 = 22\log_4 x \Rightarrow 3(\log_4 x)^2 - 22\log_4 x + 24 = 0$	M1
	$3(\log_4 x)^2 - 22\log_4 x + 24 = 0 \Rightarrow (3\log_4 x - 4)(\log_4 x - 6) = 0$	M1
	$\Rightarrow \log_4 x = \frac{4}{3}, \quad 6$	A1
	$x = 4^{\frac{4}{3}}$ or awrt 6.35 and $x = 4096$	M1A1 [7]
		Total 7 marks

8 – (4PM1/2R_Winter_2023_Q10) – *Logarithmic Functions And Indices*

(a)	$\frac{1}{243}=3^{-5}$	B1
	$9^{3y}=3^{6y}$	B1
	$\frac{9^{3y}}{243}=3^{-5}\times 3^{6y}\Rightarrow \frac{9^{3y}}{243}=3^{(6y-5)}$ *	M1A1 cso [4]
(b)	$27^{(x-2)}=3^{3(x-2)}=3^{(3x-6)}$	M1
	$6y-5=3x-6\Rightarrow(6y-3x-1=0)$	M1
	$\log_4 2=\frac{1}{2}$	B1
	$\log_{10}\sqrt{6xy}=\frac{1}{2}\log_{10}(6xy)\Rightarrow\log_{10}(6xy)=1$	M1
	$1=\log_{10}10\Rightarrow\log_{10}(6xy)=\log_{10}10\Rightarrow 6xy=10$	M1
	$3x-6y-1=0$ $6xy=10$	
	Method A	
	$6y=\frac{10}{x}\Rightarrow 3x-\frac{10}{x}-1=0\Rightarrow 3x^2-x-10=0$	M1
	$3x^2-x-10=(3x+5)(x-2)=0\Rightarrow x=2,\ -\frac{5}{3}$	M1
	$3\times 2-6y-1=0\Rightarrow 6y=5\Rightarrow y=\frac{5}{6}$	A1
	$3\times\left(-\frac{5}{3}\right)-6y-1=0\Rightarrow -6y=6\Rightarrow y=-1$	
	$x=2\quad y=\frac{5}{6}$ or $x=-\frac{5}{3}\quad y=-1$	A1 [9]
	Method B	
	$3x=\frac{5}{y}\Rightarrow\frac{5}{y}-6y-1=0\Rightarrow 6y^2+y-5=0$	M1
	$6y^2+y-5=(6y-5)(y+1)=0\Rightarrow y=\frac{5}{6},\ -1$	M1
	$3x-6\times\frac{5}{6}-1=0\Rightarrow 3x=6\Rightarrow x=2$	A1
	$3x-6\times(-1)-1=0\Rightarrow 3x=-5\Rightarrow x=-\frac{5}{3}$	
	$x=2\quad y=\frac{5}{6}$ or $x=-\frac{5}{3}\quad y=-1$	A1 [9]
		Total 13 marks

9 - (4PM1/2_Winter_2024_Q6) - *Logarithmic Functions And Indices*

(i)	$5(2\log_b 3+\log_b 3)=3$	M1
	$\left(3\log_b 3=\frac{3}{5}\right)\Rightarrow \log_b 3=\frac{1}{5}$	M1
	$b^{\frac{1}{5}}=3$	M1
	$b=243$ or 3^5	A1
	ALT	
	$5\log_b 27=3$	M1
	$\log_b 27=\frac{3}{5}$	M1
	$b^{\frac{3}{5}}=27$	M1
	$b=243$ or $b=27^{\frac{5}{3}}$	A1 [4]
(ii)	$3\log_3 x+3\frac{\log_3 27}{\log_3 x}=,28$	M1,B1
	$3(\log_3 x)^2+9=28\log_3 x$	dM1
	$3(\log_3 x)^2-28\log_3 x+9(=0)\Rightarrow(3\log_3 x-1)(\log_3 x-9)(=0)$	ddM1
	$(\log_3 x=)\frac{1}{3}$ and $(\log_3 x=)9$	A1
	$x=3^{\frac{1}{3}},3^9$ oe exact form	M1 A1 [7]
	Total 11 marks	