A LEVEL Cambridge Topical Past Papers

FURTHER MECHANICS

2020 — 2024

1 - (9231/31_Summer_2020_Q6) - *Momentum And Impulse*

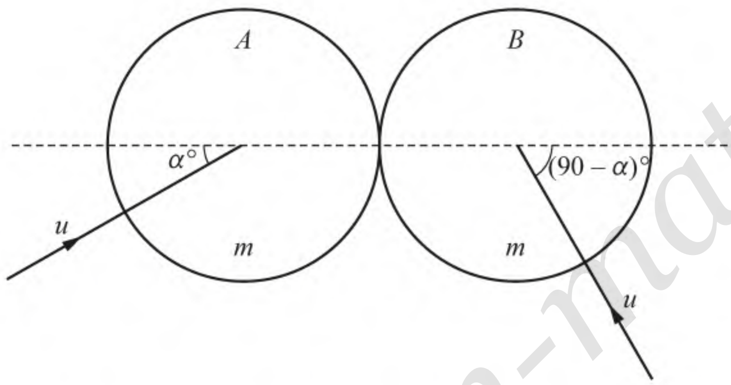
A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of the impact, the direction of motion of P is turned through 90°.

(a) Show that $\tan^2 \alpha = \frac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e. [5]

2 - (9231/33_Summer_2020_Q5) - *Momentum And Impulse*



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of $\alpha°$ with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan \alpha = \frac{1+e}{1-e}$. [4]

(b) Given that $\tan \alpha = 2$, find the speed of A after the collision. [4]

3 - (9231/31_Winter_2020_Q6) - *Momentum And Impulse*

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

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Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

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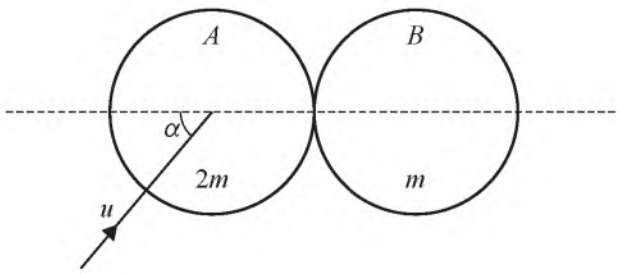
4 - (9231/32_Winter_2020_Q2) - *Momentum And Impulse*



Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision. [5]

5 - (9231/32_Winter_2020_Q4) - *Momentum And Impulse*

A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \dfrac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]

ANSWERS

1 - (9231/31_Summer_2020_Q6) - *Momentum And Impulse*

(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)	
	$y = v\cos\alpha = eu\sin\alpha$	B1
	$x = v\sin\alpha = u\cos\alpha$	B1
	Divide: $\tan\alpha = \frac{1}{e\tan\alpha}$: $\tan^2\alpha = \frac{1}{e}$.	B1
		3

(b)	$v^2 = \frac{1}{3}u^2$	B1
	$\left(\frac{u\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{3}u^2$	M1
	$(\tan\alpha)^2 = 3$	M1
	$\alpha = 60°$	A1
	$e = \frac{1}{3}$	A1
	Alternative method for **6(b)**	
	KE after impact = $\frac{1}{2}m(x^2 + y^2) = \frac{1}{2}m\left((u\cos\alpha)^2 + e^2(u\sin\alpha)^2\right)$	M1
	From **(a)** $\sin\alpha = 1/\sqrt{(1+e)}$ and $\cos\alpha = \sqrt{e}/\sqrt{(1+e)}$	B1
	KE = $\frac{1}{2}mu^2\left(\frac{e}{1+e} + \frac{e^2}{1+e}\right) = \frac{1}{2}mu^2e$	A1
	This is equal to $\frac{1}{3} \times \frac{1}{2}mu^2$ so $e = \frac{1}{3}$	M1
	$\tan\alpha = \sqrt{3}$, $\alpha = 60°$	A1
		5

2 - (9231/33_Summer_2020_Q5) - *Momentum And Impulse*

(a)	Let w be speed of A along line of centres after collision $\leftarrow \quad mw = -mu\cos\alpha + mu\sin\alpha$	M1
	$w - 0 = e(u\cos\alpha + u\sin\alpha)$	M1
	Rearrange: $\sin\alpha(u - eu) = \cos\alpha(u + eu)$	M1
	$\tan\alpha = \dfrac{1+e}{1-e}$. **AG**	A1
		4
(b)	$\tan\alpha = 2 \Rightarrow e = \dfrac{1}{3}$	B1
	$w = \dfrac{1}{3}u\left(\dfrac{1}{\sqrt{5}} + \dfrac{2}{\sqrt{5}}\right) = \dfrac{u}{\sqrt{5}}$	M1
	Speed = $\sqrt{w^2 + (u\sin\alpha)^2}$	M1
	$= \sqrt{\dfrac{u^2}{5} + \dfrac{4u^2}{5}} = u$	A1
		4

3 - (9231/31_Winter_2020_Q6) - *Momentum And Impulse*

(a)	$mu = mw + 2mv$	B1	Momentum equation (with m)
	$v - w = eu$	B1	Restitution with consistent signs
	$v = \dfrac{u}{3}(e+1)$ $w = \dfrac{u}{3}(1-2e)$	B1	Both correct.
		3	
(b)	Perpendicular to plane: $y = ev\sin\theta$ Parallel to plane: $x = v\cos\theta$	B1	Both
	Speed of $B = \sqrt{x^2+y^2} = \sqrt{v^2\left(\left(\dfrac{4}{5}\right)^2 + \left(\dfrac{2}{3}\cdot\dfrac{3}{5}\right)^2\right)}$ $\left(= \dfrac{2}{\sqrt{5}}v\right)$	M1	Speed of B
	KE of $B = \dfrac{1}{2}.2m\dfrac{4}{5}.\dfrac{u^2}{9}(e+1)^2$	M1	KE of B in terms of u. $\dfrac{1}{2}$ and $2m$ needed
	KE of $A = \dfrac{1}{2}.m.\dfrac{u^2}{9}(1-2e)^2$ So $\dfrac{1}{2}.m.\dfrac{u^2}{9}(1-2e)^2 = \dfrac{5}{32}.\dfrac{1}{2}.2m\dfrac{4}{5}.\dfrac{u^2}{9}(e+1)^2$	M1 A1	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	M1	Rearrange and simplify to quadratic
	$1 + e = \pm 2(1-2e)$ $e = \dfrac{1}{5}$, 1	A1	Both values
		7	

4 - (9231/32_Winter_2020_Q2) - *Momentum And Impulse*

Speeds v and w after collision $2mv+mw=2mu\cos\alpha$	M1	Momentum equation with m. Correct masses, allow sin instead of cos
$w-v=eu\cos\alpha$	M1	Restitution, with consistent signs
$v=\frac{1}{3}u\cos\alpha(2-e)=\frac{1}{3}u.\frac{3}{5}\left(2-\frac{1}{3}\right)=\frac{1}{3}u$	A1	
Square of speed of $A=\left(\frac{1}{3}u\right)^2+(u\sin\alpha)^2$	M1	Uses correct speed perpendicular to motion
$=\left(\frac{1}{3}u\right)^2+\left(\frac{4}{5}u\right)^2$ Speed $=\frac{13}{15}u$ $(=0.867u)$	A1	
	5	

5 - (9231/32_Winter_2020_Q4) - *Momentum And Impulse*

(a)	$\uparrow N\cos\theta=mg$	B1	
	$\leftarrow N\sin\theta=mr\sin\theta\omega^2$	B1	
	$\cos\theta=\frac{mg}{N}$ so $\cos\theta=\frac{g}{\omega^2 r}$	B1	AG
		3	
(b)	$\cos\theta=\frac{r-x}{r}=\frac{g}{\omega^2 r}$	B1	Using trig of situation: must involve x
	In new situation: $r-4x=r\times\frac{g}{4\omega^2 r}$	M1	Using new situation with $4x$ and 2ω seen
	$r-x=4(r-4x)$	M1	Combining
	$x=\frac{1}{5}r$	A1	
		4	

6 - (9231/31_Summer_2021_Q6) - *Momentum And Impulse*

(a)	Along line of centres, speeds v_1 and v_2 $mv_1+mv_2=mu\cos\alpha-mu\cos\beta$	M1	Momentum (condone missing masses).
	$v_2-v_1=eu(\cos\beta+\cos\alpha)$	M1	Restitution.
	Both correct, masses seen.	A1	
	$v_1=0$ so A has no speed along line of centres: moves perpendicular to line of centres	A1	AG.
		4	
(b)	$(v_2=\frac{1}{2}u\cos\alpha=u\cos\beta$) KE of B after collision is $\frac{1}{2}m\left(v_2^2+(u\sin\beta)^2\right)$ KE of A after collision $=\frac{1}{2}m(u\sin\alpha)^2$	M1	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	M1	
	Simplify to equation in $\sin\alpha$	M1	
	$\sin\alpha=\frac{1}{\sqrt{2}}$, $\alpha=45°$	A1	
		4	