A LEVEL Cambridge Topical Past Papers

STATISTICS 2

2020 — 2024

1 - (9709/61_Summer_2020_Q4) - *Hypothesis Testing Using Binomial Distribution*

A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $\text{Var}(X) = 2$. [1]

Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. [4]

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(c) State what is meant by a Type II error in this context. [1]

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2 - (9709/62_Summer_2020_Q2) - *Hypothesis Testing Using Binomial Distribution*

A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are Grade A. Following a change in growing conditions at the farm, the shop management plan to carry out a hypothesis test to find out whether the proportion of Grade A apples has increased. They select 25 apples at random. If the number of Grade A apples is more than 3 they will conclude that the proportion has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the probability of a Type I error. [3]

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In fact 2 of the 25 apples were Grade A.

(c) Which of the errors, Type I or Type II, is possible? Justify your answer. [2]

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3 - (9709/62_Summer_2020_Q3) - *Hypothesis Testing Using Binomial Distribution, Poisson Distribution*

In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution of X, including the values of any parameters. [1]

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(ii) State an appropriate approximating distribution for X, including the values of any parameters.

Justify your choice of approximating distribution. [3]

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(iii) Use your approximating distribution to find $P(X > 2)$. [2]

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(b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n. [3]

4 - (9709/63_Summer_2020_Q5) - *Hypothesis Testing Using Binomial Distribution*

Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p. Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p. [4]

(b) Sunita believes that the die is fair. Use your answer to part **(a)** to comment on her belief. [1]

(c) Sunita uses the result of her 500 throws to calculate an $\alpha\%$ confidence interval for p. This interval has width 0.04.

Find the value of α. [5]

ANSWERS

1 - (9709/61_Summer_2020_Q4) - *Hypothesis Testing Using Binomial Distribution*

(a)	$(1^2+2^2+3^2+4^2+5^2)\div 5-3^2$ (= 2 **AG**)	**B1**
		1
(b)	N(3, 2)	**M1**
	$\frac{2.6-\text{"3"}}{\sqrt{\frac{2}{40}}}(=-1.789)$	**M1**
	Φ("–1.789") = 1 – Φ("1.789")	**M1**
	0.0367 to 0.0368	**A1**
		4
(c)	Concluding that spinner is unbiased when it is biased	**B1**
		1

2 - (9709/62_Summer_2020_Q2) - *Hypothesis Testing Using Binomial Distribution*

(a)	H_0: Proportion = 0.05 H_1: Proportion > 0.05	**B1**
		1
(b)	$1-(0.95^{25}+25\times 0.95^{24}\times 0.05+{}^{25}C_2\times 0.95^{23}\times 0.05^2+{}^{25}C_3\times 0.95^{22}\times 0.05^3)$	**M1**
	Completely correct expression	**A1**
	0.0341	**A1**
		3
(c)	Type II	**B1**
	Will conclude proportion not increased	**B1**
		2

3 - (9709/62_Summer_2020_Q3) - *Hypothesis Testing Using Binomial Distribution, Poisson Distribution*

(a)(i)	B(3600, 0.0012)	**B1**
		1
(a)(ii)	Po(4.32) (**B1** for Po. **B1** for $\lambda = 4.32$)	**B2**
	$n = 3600$ which is large, $p = 0.12$ which is small and $np = 4.32$ which is < 5	**B1**
		3
(a)(iii)	$1-e^{-4.32}\left(1+4.32+\frac{4.32^2}{2}\right)$	**M1**
	0.805 (3 sf)	**A1**
		2
(b)	$e^{-\lambda} > 0.1$	**M1**
	$(-\lambda > \ln 0.1)$ $(\lambda < \ln 10)$ $0.0012n < \ln 10$	**A1**
	$(n < 1918.8)$ largest n is 1918	**A1**
		3

4 - (9709/63_Summer_2020_Q5) - *Hypothesis Testing Using Binomial Distribution*

(a)	$p = \frac{70}{500}$ or 0.14	B1
	$z = 2.576$	B1
	$"0.14" \pm z \times \sqrt{\frac{"0.14"(1-"0.14")}{500}}$	M1
	0.100 to 0.180	A1
		4

(b)	0.1666... is within confidence interval Belief supported or justified	B1
		1
(c)	$z \times \sqrt{\frac{"0.14"(1-"0.14")}{500}} = 0.02$	M1
	$z = 1.289$	A1
	$\Phi('1.289') = 0.9013$	M1
	$\alpha = '0.9013' - (1 - '0.9013')$	M1
	80.3% (3 sf)	A1
		5

5 - (9709/61_Winter_2020_Q5) - *Poisson Distribution, Hypothesis Testing Using Binomial Distribution*

(a)	$\sqrt{2.1}$ or 1.45 (3 sf)	B1	
		1	
(b)	$\lambda = 4.2$	B1	
	$1 - e^{-4.2}(1 + 4.2)$	M1	$1 - P(X \leqslant 1)$ any λ, allow one end error.
	$= 0.922$ (3 sf)	A1	
		3	
(c)	$\lambda = 6.3$ $e^{-6.3}\left(\frac{6.3^5}{5!} + \frac{6.3^6}{6!} + \frac{6.3^7}{7!}\right)$	M1	$P(X = 5, 6, 7)$ any λ, allow one end error.
	$= 0.455$ (3 sf)	A1	
		2	
(d)	$H_0: \lambda = 6.3$ $H_1: \lambda < 6.3$	B1	Accept μ, accept 2.1 (per week)
	$P(X \leqslant 2) = e^{-6.3}\left(1 + 6.3 + \frac{6.3^2}{2!}\right)$	M1	
	$= 0.0498$ or 0.0499	A1	Accept 0.0499
	'0.0498' < 0.1	M1	For valid comparison. For CV method the comparison can be '2 lies in CR of $X \leqslant 2$'
	There is evidence that mean number of absences has decreased.	A1 FT	In context, not definite, e.g. not 'Mean number of absences has decreased.' No contradictions.
		5	