

Cambridge A LEVEL
Topical Past Papers

PURE MATHEMATICS 1

9709

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CHAPTER 1

Coordinates Geometry

1 - (9709/11_Summer_2017_Q5)

ANSWER

The equation of a curve is $y = 2 \cos x$.

- (i) Sketch the graph of $y = 2 \cos x$ for $-\pi \leq x \leq \pi$, stating the coordinates of the point of intersection with the y -axis. [2]

Points P and Q lie on the curve and have x -coordinates of $\frac{1}{3}\pi$ and π respectively.

- (ii) Find the length of PQ correct to 1 decimal place. [2]

The line through P and Q meets the x -axis at $H(h, 0)$ and the y -axis at $K(0, k)$.

- (iii) Show that $h = \frac{5}{9}\pi$ and find the value of k . [3]

2 - (9709/12_Summer_2017_Q2)

ANSWER

The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

- (i) Find the equation of AB . [3]

- (ii) Find the coordinates of B . [3]

3 - (9709/13_Summer_2017_Q8)

ANSWER

$A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

- (i) Find an expression for b in terms of a . [2]

- (ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P . [6]

4 - (9709/11_Winter_2017_Q6)

ANSWER

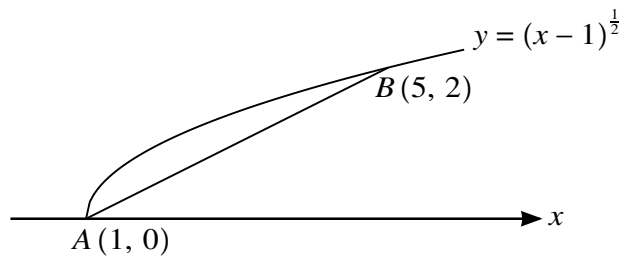
The points $A(1, 1)$ and $B(5, 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

- (i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D .

- (ii) Find, by calculation, the distance CD , giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

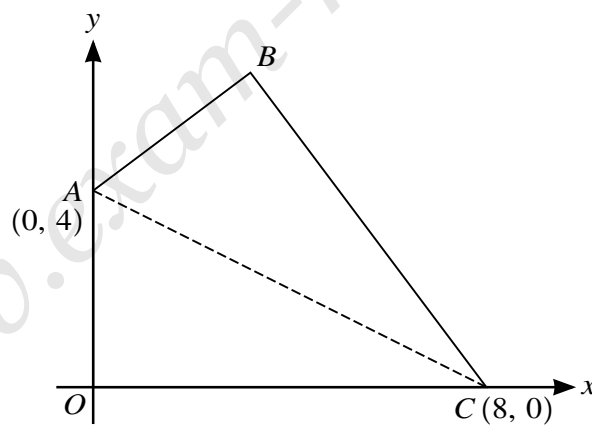
5 - (9709/13_Winter_2017_Q11)



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A(1, 0)$ and $B(5, 2)$ lying on the curve.

- (i) Find the equation of the line AB , giving your answer in the form $y = mx + c$. [2]
- (ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB . [5]
- (iii) Find the perpendicular distance between the line AB and the tangent parallel to AB . Give your answer correct to 2 decimal places. [3]

6 - (9709/11_Summer_2018_Q5)



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are $(0, 4)$ and $(8, 0)$ respectively and O is the origin.

- (i) Find the equations of AC and OB . [4]
- (ii) Find, by calculation, the coordinates of B . [3]

7 - (9709/12_Summer_2018_Q8)

ANSWER

Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k . [7]

8 - (9709/13_Summer_2018_Q6)

ANSWER

The coordinates of points A and B are $(-3k - 1, k + 3)$ and $(k + 3, 3k + 5)$ respectively, where k is a constant ($k \neq -1$).

(i) Find and simplify the gradient of AB , showing that it is independent of k . [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB . [5]

9 - (9709/11_Winter_2018_Q3)

ANSWER

Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB . [2]

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a . [3]

10 - (9709/12_Winter_2018_Q10)

ANSWER

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B .

(ii) Find the coordinates of A and B . [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B . [3]

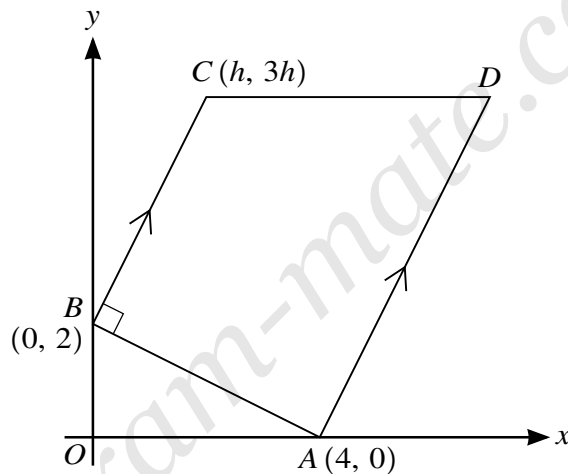
11 - (9709/13_Winter_2018_Q4)



Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x -axis at C .

- (i) Find the equation of BC and the x -coordinate of C . [4]
 (ii) Find the distance AC , giving your answer correct to 3 decimal places. [2]

12 - (9709/11_Summer_2019_Q4)



The diagram shows a trapezium $ABCD$ in which the coordinates of A , B and C are $(4, 0)$, $(0, 2)$ and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90^\circ$ and CD is parallel to the x -axis.

- (i) Find, by calculation, the value of h . [3]
 (ii) Hence find the coordinates of D . [3]

13 - (9709/12_Summer_2019_Q2)



Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y -axis at the point C . Find the coordinates of C . [5]

14 - (9709/13_Summer_2019_Q7)

ANSWER

The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB . A point C has coordinates (x, y) , where x and y are variables.

- (i) State the coordinates of D . [1]
- (ii) It is given that $CD^2 = 20$. Write down an equation relating x and y . [1]
- (iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]
- (iv) Using the results from parts (ii) and (iii), and showing all necessary working, find the possible coordinates of C . [4]

15 - (9709/12_Winter_2019_Q2)

ANSWER

The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

16 - (9709/11_Summer_2020_Q10)

ANSWER

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

- (a) Find the equation of the circle, C , for which AB is a diameter. [4]
- (b) Find the equation of the tangent, T , to circle C at the point B . [4]
- (c) Find the equation of the circle which is the reflection of circle C in the line T . [3]

1 - (9709/11_Summer_2017_Q5)



	$y = 2\cos x$	
(i)		
	Total:	2
(ii)	$P\left(\frac{\pi}{3}, 1\right) Q(\pi, -2)$	
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	
	Total:	2
(iii)	Eqn of PQ $y - 1 = -\frac{9}{2\pi}\left(x - \frac{\pi}{3}\right)$	
	If $y = 0 \rightarrow h = \frac{5\pi}{9}$	
	If $x = 0 \rightarrow k = \frac{5}{2}$,	
	Total:	3

2 - (9709/12_Summer_2017_Q2)



(i)	Gradient = 1.5 Gradient of perpendicular = $-\frac{2}{3}$	
	Equation of AB is $y - 6 = -\frac{2}{3}(x + 2)$ Or $3y + 2x = 14$ oe	
	Total:	3
(ii)	Simultaneous eq \rightarrow Midpoint $(1, 4)$	
	Use of midpoint or vectors $\rightarrow B(4, 2)$	
	Total:	3

3 - (9709/13_Summer_2017_Q8)



(i)	$(b-1)/(a+1) = 2$	
	$b = 2a + 3$ CAO	
	Total:	2
(ii)	$AB^2 = 11^2 + 2^2 = 125$ oe	
	$(a+1)^2 + (b-1)^2 = 125$	
	$(a+1)^2 + (2a+2)^2 = 125$	
	$(5)(a^2 + 2a - 24) = 0 \rightarrow \text{eg } (a-4)(a+6) = 0$	
	$a = 4$ or -6	
	$b = 11$ or -9	
	Total:	6

4 - (9709/11_Winter_2017_Q6)



(i)	Mid-point of $AB = (3, 5)$	
	Gradient of $AB = 2$	
	Eqn of perp. bisector is $y - 5 = -\frac{1}{2}(x - 3) \rightarrow 2y = 13 - x$	
		4
(ii)	$-3x + 39 = 5x^2 - 18x + 19 \rightarrow (5)(x^2 - 3x - 4) = 0$	
	$x = 4$ or -1	
	$y = 4\frac{1}{2}$ or 7	
	$CD^2 = 5^2 + 2\frac{1}{2}^2 \rightarrow CD = \sqrt{\frac{125}{4}}$	
		5

5 - (9709/13_Winter_2017_Q11)



(i)	Gradient of $AB = \frac{1}{2}$	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	
		2
(ii)	$\frac{dy}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}}$	
	$\frac{1}{2}(x-1)^{-\frac{1}{2}} = \frac{1}{2}$. Equate <i>their</i> $\frac{dy}{dx}$ to <i>their</i> $\frac{1}{2}$	
	$x = 2, y = 1$	
	$y - 1 = \frac{1}{2}(x - 2)$ (thro' <i>their</i> (2,1) & <i>their</i> $\frac{1}{2}$) $\rightarrow y = \frac{1}{2}x$	
		5

(ii)	<i>EITHER:</i> $\sin \theta = \frac{d}{1} \rightarrow d = \sin \theta$	
	gradient of $AB = \frac{1}{2} \Rightarrow \tan \theta = \frac{1}{2} \Rightarrow \theta = 26.5(7)^\circ$	
	$d = \sin 26.5(7)^\circ = 0.45$ (or $\frac{1}{\sqrt{5}}$)	
	<i>OR1:</i> Perpendicular through O has equation $y = -2x$	
	Intersection with AB : $-2x = \frac{1}{2}x - \frac{1}{2} \rightarrow \left(\frac{1}{5}, \frac{-2}{5}\right)$	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	
	<i>OR2:</i> Perpendicular through $(2, 1)$ has equation $y = -2x + 5$	
	Intersection with AB : $-2x + 5 = \frac{1}{2}x - \frac{1}{2} \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$)	
(iii)	<i>OR3:</i> ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \rightarrow d = \frac{1}{\sqrt{5}}$	
		3

6 - (9709/11_Summer_2018_Q5)



5(i)	Eqn of AC $y = -\frac{1}{2}x + 4$ (gradient must be $\Delta y / \Delta x$)	
	Gradient of $OB = 2 \rightarrow y = 2x$ (If y missing only penalise once)	
		4
5(ii)	Simultaneous equations $\rightarrow ((1.6, 3.2))$	
	This is mid-point of OB . $\rightarrow B (3.2, 6.4)$	
	or	
	Let coordinates of $B (h, k)$ $OA = AB \rightarrow h^2 = 8k - k^2$ $OC = BC \rightarrow k^2 = 16h - h^2 \rightarrow (3.2, 6.4)$	
	or	
	gradients $(\frac{k-4}{h} \times \frac{k}{h-8} = -1)$	
	or	
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$	
	3	

7 - (9709/12_Summer_2018_Q8)



EITHER	
Gradient of bisector = $-\frac{3}{2}$	
gradient $AB = \frac{5h-h}{4h+6-h}$	
Either $\frac{5h-h}{4h+6-h} = \frac{2}{3}$ or $-\frac{4h+6-h}{5h-h} = -\frac{3}{2}$	
OR	
Gradient of bisector = $-\frac{3}{2}$	
Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x + \frac{h}{3} = y$ oe	
Substitute co-ordinates of one of the other points	
$h = 2$	
Midpoint is $\left(\frac{5h+6}{2}, 3h\right)$ or $(8, 6)$	
Uses midpoint and 'their h ' with $3x + 2y = k$	
$\rightarrow k = 36$ soi	
	7

8 - (9709/13_Summer_2018_Q6)



(i)	Gradient, m , of $AB = \frac{3k+5-(k+3)}{k+3-(-3k-1)}$ OE $\left(= \frac{2k+2}{4k+4} \right) = \frac{1}{2}$	
		2