Cambridge IGCSE

ADDITIONAL MATHEMATICS

0606 P2

2017 - 2024
QUESTIONS + ANSWERS

CHAPTER 2

Intersection Points

1 -(0606/21_Summer_2017_Q9) ANSWER

The curve $3x^2 + xy - y^2 + 4y - 3 = 0$ and the line $y = 2(1 - x)$ intersect at the points A and B.

(i) Find the coordinates of A and of B. [5]

(ii) Find the equation of the perpendicular bisector of the line AB, giving your answer in the form $ax + by = c$, where a, b and c are integers. [4]

2 -(0606/21_Winter_2017_Q4) ANSWER

Solve the following simultaneous equations for x and y, giving each answer in its simplest surd form.

$$\sqrt{3}x + y = 4$$

$$x - 2y = 5\sqrt{3}$$ [5]

3 -(0606/22_Summer_2018_Q4) ANSWER

Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\frac{x^2}{4} + \frac{y^2}{9} = 5$. [5]

4 -(0606/21_Winter_2018_Q10) ANSWER

The line $y = 12 - 2x$ is a tangent to two curves. Each curve has an equation of the form $y = k + 6 + kx - x^2$, where k is a constant.

(i) Find the two values of k. [5]

The line $y = 12 - 2x$ is a tangent to one curve at the point A and the other curve at the point B.

(ii) Find the coordinates of A and of B. [3]

(iii) Find the equation of the perpendicular bisector of AB. [3]

5 -(0606/22_Winter_2018_Q10) ANSWER

Two lines are tangents to the curve $y = 12 - 4x - x^2$. The equation of each tangent is of the form $y = 2k + 1 - kx$, where k is a constant.

(i) Find the two possible values of k. [5]

(ii) Find the coordinates of the point of intersection of the two tangents. [4]

6 -(0606/23_Winter_2018_Q11) ANSWER

A line with equation $y = -5x + k + 5$ is a tangent to a curve with equation $y = 7 - kx - x^2$.

(i) Find the two possible values of k. [5]

(ii) Find, for **each** of your values of k,

- the equation of the tangent
- the equation of the curve
- the coordinates of the point of contact of the tangent and the curve. [5]

(iii) Find the distance between the two points of contact. [2]

7 -(0606/21_Winter_2019_Q4) ANSWER

Do not use a calculator in this question.

Solve the following simultaneous equations, giving your answers for both x and y in the form $a+b\sqrt{2}$, where a and b are integers.

$$2x+y=5$$

$$3x-\sqrt{2}y=7$$

[5]

8 -(0606/22_Winter_2019_Q4) ANSWER

Find the values of k for which the line $y=kx+3$ does not meet the curve $y=x^2+5x+12$. [5]

9 -(0606/21_Summer_2020_Q6) ANSWER

Find the values of k for which the line $y=kx-7$ and the curve $y=3x^2+8x+5$ do not intersect. [6]

10 -(0606/22_Summer_2020_Q3) ANSWER

Find the values of k for which the line $y=x-3$ intersects the curve $y=k^2x^2+5kx+1$ at two distinct points. [6]

11 -(0606/23_Summer_2020_Q2) ANSWER

Find the set of values of k for which $4x^2-4kx+2k+3=0$ has no real roots. [5]

12 -(0606/21_Winter_2020_Q2) ANSWER

Find the coordinates of the points of intersection of the curve $x^2+xy=9$ and the line $y=\frac{2}{3}x-2$. [5]

13 -(0606/23_Winter_2020_Q2) ANSWER

Solve the simultaneous equations.

$$x^2+3xy=4$$

$$2x+5y=4$$

[5]

1 -(0606/21_Summer_2017_Q9) QUESTION

(i)	Substitution of $y = 2(1-x)$	M1	Must be attempt at full substitution. Condone one sign error in substitution. Condone omission of = 0 or incorrect rhs
	$-3x^2 + 2x + 1 = 0$ oe $(3x^2 - 2x - 1 = 0)$	A1	Terms collected
	Solving *their* quadratic found from eliminating one variable $(3x+1)(1-x)$ or $(3x+1)(x-1)$	M1	can be implied by a correct pair of x values
	$\left(-\frac{1}{3}, \frac{8}{3}\right)$ oe and $(1, 0)$ oe isw nfww	A2	**A1** for each or **A1** for a correct pair of x-coordinates or a correct pair of y-coordinates
(ii)	$[m =]\frac{1}{2}$ cao	B1	
	$\left(\frac{1}{3}, \frac{4}{3}\right)$	B1	FT
	$y - their\frac{4}{3} = their\frac{1}{2}\left(x - their\frac{1}{3}\right)$	M1	or $y = their\frac{1}{2}x + c$ and substitutes their midpoint and reaches $c = \ldots$
	$6y - 3x = 7$	A1	allow any equivalent form with integer coeffs/constant

2 - (0606/21_Winter_2017_Q4)

	$x-2\left(4-\sqrt{3}x\right)=5\sqrt{3}$	**M1**	Eliminate y
	$x=\dfrac{5\sqrt{3}+8}{2\sqrt{3}+1}$	**A1**	
	$x=\dfrac{\left(5\sqrt{3}+8\right)\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$x=2+\sqrt{3}$	**A1**	
	$y=1-2\sqrt{3}$	**A1**	
	<u>Alternative method</u> $\sqrt{3}\left(5\sqrt{3}+2y\right)+y=4$	**M1**	Eliminate x
	$y=\dfrac{-11}{\left(2\sqrt{3}+1\right)}$	**A1**	
	$y=\dfrac{-11\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$y=1-2\sqrt{3}$	**A1**	
	$x=2+\sqrt{3}$	**A1**	

3 - (0606/22_Summer_2018_Q4)

	Eliminates one of the unknowns	**M1**	
	Simplifies to a correct 3-term quadratic: $2x^2 + 4x - 16$ [= 0] oe or $2y^2 - 6y - 36$ [= 0] oe	**A1**	
	Factorises or solves $(x+4)(x-2)=0$ oe or $(y+3)(y-6)=0$ oe	**M1**	**FT** *their* 3-term quadratic in x or y;
	(2, 6) and (–4, –3) oe	**A2**	Not from wrong working **A1** for either (2, 6) or (–4, –3) or **A1** for $x = 2$ and $x = -4$ or $y = 6$ and $y = -3$

4 - (0606/21_Winter_2018_Q10) QUESTION

(i)	$12-2x=k+6+kx-x^2$ $\rightarrow x^2-(2+k)x+6-k=0$	M1	* Equate and collect terms
	$b^2-4ac=0$ $\rightarrow (2+k)^2=4(6-k)$	M1	**Dep***
	$k^2+8k-20=0$	A1	
	$(k+10)(k-2)=0$	M1	
	$k=-10$ or 2	A1	
(ii)	$(-4,\ 20)$ and $(2,\ 8)$	3	**M1** Insert values of k in equations and solve for x **A1** $x^2+8x+16=0 \rightarrow x=-4$ $\rightarrow y=20$ **A1** $x^2-4x+4=0$ $\rightarrow x=2 \rightarrow y=8$
(iii)	Grad of perpendicular $=\frac{1}{2}$	B1	
	Midpoint $(-1\ ,14)$	B1	**FT**
	Eqn $\frac{y-14}{x+1}=\frac{1}{2} \rightarrow y=\frac{1}{2}x+14.5$	B1	**FT**

5 -(0606/22_Winter_2018_Q10) QUESTION

(i)	$2k+1-kx=12-4x-x^2$ $x^2+4x-kx+2k-12+1$	M1	*
	b^2-4ac $\rightarrow(4-k)^2-4(2k-11)$	M1	Dep*
	$k^2-16k+60$	A1	
	$(k-6)(k-10)$	M1	
	$k=6 \text{ or } 10$	A1	
	OR		
	$k=4+2x$	M1	*
	$-4x-2x^2+8+4x+1=12-4x-x^2$ or $2k+1-k\left(\frac{k-4}{2}\right)=12-2(k-4)-\left(\frac{k-4}{2}\right)^2$	M1	Dep*
	x^2-4x+3 or $k^2-16k+60$	A1	
	$(x-1)(x-3)$ or $(k-6)(k-10)$	M1	
	$x=1 \text{ or } x=3 \rightarrow k= 6 \text{ or } 10$	A1	
(ii)	$k=6\rightarrow[y]=13-6x$	B1	FT
	$k=10\rightarrow[y]=21-10x$	B1	FT
		M1	solve
	$x=2, y=1.$	2	cao

6 -(0606/23_Winter_2018_Q11) QUESTION

(i)	$-5x+k+5=7-kx-x^2$	**M1**	*
	$b^2-4ac\ (=0) \ \rightarrow\ (k-5)^2-4(k-2)\ (=0)$	**M1**	**Dep***
	$k^2-14k+33 \quad (=0)$	**A1**	
	$(k-11)(k-3)\ \ (=0)$	**M1**	**Dep dep** * solve quadratic in k
	$k=11$ and $k=3$	**A1**	
(ii)	$y=-5x+16$ and $y=7-11x-x^2$ $y=-5x+8$ and $y=7-3x-x^2$	**B2**	**FT** *their k* **B1** for any two correct
	solve one tangent/curve pair for one variable from quadratic equation with repeated root	**M1**	
	$(-3, 31)$ and $(1, 3)$	**A2**	**A1** for one correct point or two correct x values
(iii)	find distance between any two points found in **(ii)**	**M1**	
	$\sqrt{800}$ oe	**A1**	

7 -(0606/21_Winter_2019_Q4) QUESTION

	Eliminate x or y	**M1**	
	$x=\frac{7+5\sqrt{2}}{3+2\sqrt{2}}$ or $y=\frac{1}{3+2\sqrt{2}}$	**A1**	
	Multiply numerator and denominator by $3-2\sqrt{2}$	**M1**	
	$x=1+\sqrt{2}$	**A1**	
	$y=3-2\sqrt{2}$	**A1**	

8 - (0606/22_Winter_2019_Q4)

	$kx + 3 = x^2 + 5x + 12$ $\rightarrow x^2 + (5 - k)x + 9 (= 0)$	**M1**	Equate and attempt to simplify to all terms on one side.
	Use discriminant of *their* quadratic.	**M1**	**dep**
	$(5 - k)^2 - 36$ oe	**A1**	Unsimplified
	$k = -1$ and 11	**A1**	Both boundary values
	$-1 < k < 11$	**A1**	Must be in terms of k.
	OR $2x + 5 \sim k$	**M1**	Connect gradients of line and curve
	$y = (2x + 5)x + 3 \rightarrow$ $2x^2 + 5x + 3 = x^2 + 5x + 12$	**M1**	Eliminate k and y.
	$x^2 = 9 \rightarrow x = \pm 3$	**A1**	
	$k = 11$ or $k = -1$	**A1**	
	$-1 < k < 11$	**A1**	

9 - (0606/21_Summer_2020_Q6)

	$x^2 + 8x + 5 = kx - 7$	**M1**	
	$3x^2 + (8 - k)x + 12 [= 0]$ soi	**A1**	
	$(8 - k)^2 - 4(3)(12)$	**M1**	
	$k^2 - 16k - 80 * 0$	**M1**	
	Critical values: -4 and 20 soi	**A1**	
	$-4 < k < 20$	**A1**	Alternative method: **M1** for $k = 6x + 8$ oe **M1** for $y = (6x + 8)x - 7$ **M1** for $3x^2 + 8x + 5 = (6x + 8)x - 7$ **A1** for $x = \pm 2$ **A1** for $k = -4$, $k = 20$ **A1** for $-4 < k < 20$

10 -(0606/22_Summer_2020_Q3)

	$x - 3 = k^2x^2 + 5kx + 1$	M1	
	$k^2x^2 + (5k - 1)x + 4 = 0$ soi	A1	
	$(5k-1)^2 - 4(k^2)(4)$	M1	
	$9k^2 - 10k + 1 * 0$	M1	
	Critical values: $\frac{1}{9}$ and 1 soi	A1	
	$k < \frac{1}{9}$ or $k > 1$	A1	

11 -(0606/23_Summer_2020_Q2)

	Uses $b^2 - 4ac$ $(-4k)^2 - 4(4)(2k+3)$ soi	M1	
	Correctly simplifies $16k^2 - 32k - 48$	A1	**FT** provided of equivalent difficulty
	$16(k+1)(k-3)$ oe	M1	
	CV $-1, 3$	A1	
	$-1 < k < 3$	A1	**FT** *their* lower CV $< k <$ *their* upper CV

12 -(0606/21_Winter_2020_Q2)

	$x^2 + x\left(\frac{2}{3}x - 2\right) = 9$	M1	Eliminate y
	$5x^2 - 6x - 27 = 0$	A1	
	$(x-3)(5x+9) = 0$	M1	Factorise or formula
	(3, 0)	A1	Or both x values
	$\left(-\frac{9}{5}, -\frac{16}{5}\right)$	A1	

13 - (0606/23_Winter_2020_Q2) QUESTION

	$x^2 + 3x\left(\frac{4-2x}{5}\right) = 4$	M1	eliminate x or y
	$x^2 - 12x + 20 \quad (= 0)$	A1	3 terms on one side if eliminating y $5y^2 + 16y \quad (= 0)$ if eliminating x
	$(x-2)(x-10) \quad (= 0)$	M1	or $y(5y+16) \quad (= 0)$
	$x = 2$ or $x = 10$ nfww	A1	or correct pair
	$y = 0$ or $y = -\frac{16}{5}$ nfww	A1	

14 - (0606/23_Winter_2020_Q3) QUESTION

	$(k+9)^2 - 4\times 9 \ (> 0)$	M1	use $b^2 - 4ac$
	$k^2 + 18k + 45 \quad (> 0)$	A1	
	$k = -15 \quad k = -3$	A1	
	$k < -15$ or $k > -3$ no isw mark final answer	A1	not 'and' A0 if combined as one statement