Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME												
CENTRE NUMBER							CANDIDATE NUMBER					

3353608262

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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[Turn over

1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$(3+x)(1-2x)^{1/2} = (3+x)\left(1 - \frac{1}{2}(2x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2}(2x)^2 + \dots\right)$

$= (3+x)\left(1 - x - \frac{1}{2}x^2 + \dots\right)$

$= 3 - 3x - \frac{3}{2}x^2 + x - x^2 + \dots$

$= 3 - 2x - \frac{5}{2}x^2 + \dots$

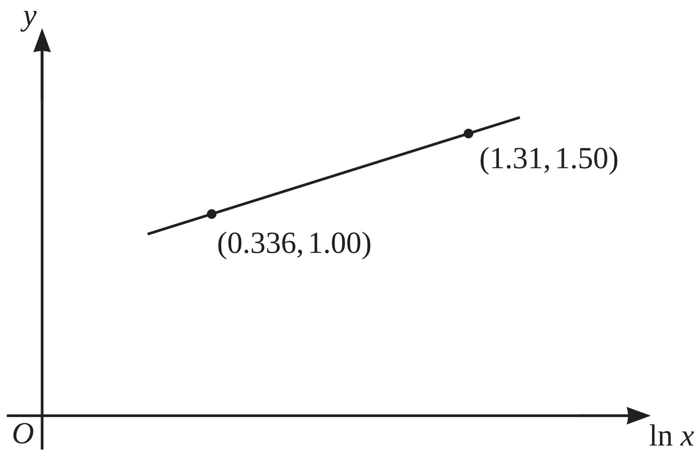
2 Solve the equation $\ln(x - 5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

$\ln(x-5) = \ln e^7 - \ln x \rightarrow \ln(x-5) = \ln \frac{e^7}{x}$

$\rightarrow x-5 = \frac{e^7}{x} \rightarrow x^2 - 5x - e^7 = 0$

$\rightarrow x = \frac{5 \pm \sqrt{25+4e^7}}{2} \rightarrow x = 35.71$ ✓

3



The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points (0.336, 1.00) and (1.31, 1.50), as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

$a^y = bx \rightarrow \ln a^y = \ln(bx) \rightarrow y \ln a = \ln b + \ln x$

$\rightarrow y = \frac{1}{\ln a} \ln x + \frac{\ln b}{\ln a}$ (0.336, 1.00)

↳ gradient ↳ y-int (1.31, 1.50)

gradient $= \frac{1.50 - 1.00}{1.31 - 0.336} = 0.5133 = \frac{1}{\ln a}$

$\rightarrow \ln a = 1.948 \rightarrow a = e^{1.948} \rightarrow a = 7$

Sub (0.336, 1.00) $\rightarrow 1 = 0.5133(0.336) + \ln b(0.5133)$

$\rightarrow b = 5$ #

4 The complex number u is given by $u = -1 - \mathrm{i}\sqrt{3}$.

(a) Express u in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [2]

$u = -1 - i\sqrt{3} \rightarrow |u| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

$\theta = \arg(u) = \tan^{-1}(-\sqrt{3}) = -\frac{2}{3}\pi$

$u = 2\left(\cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)\right)$

$= 2\left(\cos\frac{2}{3}\pi - i\sin\frac{2}{3}\pi\right)$ #

The complex number v is given by $v = 5\left(\cos\frac{1}{6}\pi + \mathrm{i}\sin\frac{1}{6}\pi\right)$.

(b) Express the complex number $\frac{v}{u}$ in the form $re^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

$u = 2e^{-\frac{2}{3}\pi}$ $\quad v = 5e^{\frac{1}{6}\pi}$

$\frac{v}{u} = \frac{r_2}{r_1} e^{\theta_2 - \theta_1} = \frac{5}{2} e^{\frac{\pi}{6} - -\frac{2\pi}{3}} = \frac{5}{2} e^{5\pi/6}$ #