



Cambridge International AS & A Level

CANDIDATE
NAME

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NUMBER

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places.

[3]

$$\text{let } e^{2x} = u \rightarrow e^{-2x} = \frac{1}{u}$$

$$e^{2x} > 0 \rightarrow u > 0$$

$$3u - \frac{4}{u} = 5 \xrightarrow{\times u} 3u^2 - 4 = 5u \rightarrow 3u^2 - 5u - 4 = 0$$

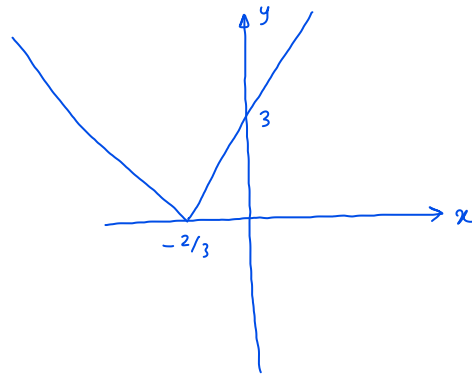
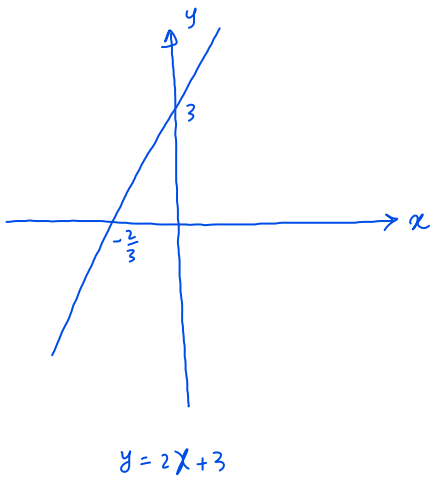
$$u = \frac{5 \pm \sqrt{25 + 48}}{6} \rightarrow \begin{cases} u = \frac{5 + \sqrt{73}}{6} \\ u = \frac{5 - \sqrt{73}}{6} < 0 \text{ rejected} \end{cases}$$

$$\rightarrow e^{2x} = \frac{5 + \sqrt{73}}{6} \rightarrow 2x = \ln\left(\frac{5 + \sqrt{73}}{6}\right) \rightarrow x = \frac{1}{2} \ln\left(\frac{5 + \sqrt{73}}{6}\right)$$

$$\rightarrow x \approx 0.407 \quad *$$

2 (a) Sketch the graph of $y = |2x + 3|$.

[1]



(b) Solve the inequality $3x + 8 > |2x + 3|$.

[3]

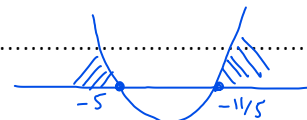
$$3x + 8 > 0 \rightarrow x > -8/3$$

$$\text{Power 2: } (3x + 8)^2 > |2x + 3|^2 \rightarrow (3x + 8)^2 - (2x + 3)^2 > 0$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$\rightarrow [(3x + 8) + (2x + 3)][(3x + 8) - (2x + 3)] > 0$$

$$\rightarrow [5x + 11][x + 5] > 0$$



$$x < -5 \quad \text{OR} \quad x > -11/5$$

$$\text{answer: } x > -11/5$$

3 Find the coefficient of x^3 in the binomial expansion of $(3+x)\sqrt{1+4x}$.

[4]

$$(1+4x)^{1/2} = 1 + \frac{1}{2}(4x) + \frac{\frac{1}{2}(-1/2)}{2}(4x)^2 + \frac{\frac{1}{2}(-1/2)(-3/2)}{6}(4x)^3 + \dots$$

$$= 1 + 2x - 2x^2 + 4x^3 - \dots$$

$$(3+x)(1+2x-2x^2+4x^3-\dots)$$

$$12x^3 - 2x^3 = 10x^3$$

↳ answer = 10 ✱

- 4 (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0. \quad [2]$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\text{Sub : } 2 \sin \theta \cos \theta + \cos^2 \theta - \sin^2 \theta = 2 \sin^2 \theta$$

$$\rightarrow \cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0 \quad \neq$$

- (b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$ for $0^\circ < \theta < 180^\circ$. [4]

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0 \quad \div \cos^2 \theta \rightarrow \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{2 \sin \theta \cos \theta}{\cos^2 \theta} - \frac{3 \sin^2 \theta}{\cos^2 \theta} = 0$$

$$\rightarrow 1 + 2 \tan \theta - 3 \tan^2 \theta = 0 \rightarrow 3 \tan^2 \theta - 2 \tan \theta - 1 = 0$$

$$\rightarrow \begin{cases} \tan \theta = 1 \rightarrow \theta = 45^\circ \\ \tan \theta = -1/3 \rightarrow \theta = \tan^{-1}(-1/3) \rightarrow \theta = -18.43 \quad (0 < \theta < 180^\circ) \end{cases}$$

$$\downarrow$$

$$+180^\circ = 161.6^\circ$$