

IB Diploma

MATHEMATICS AA

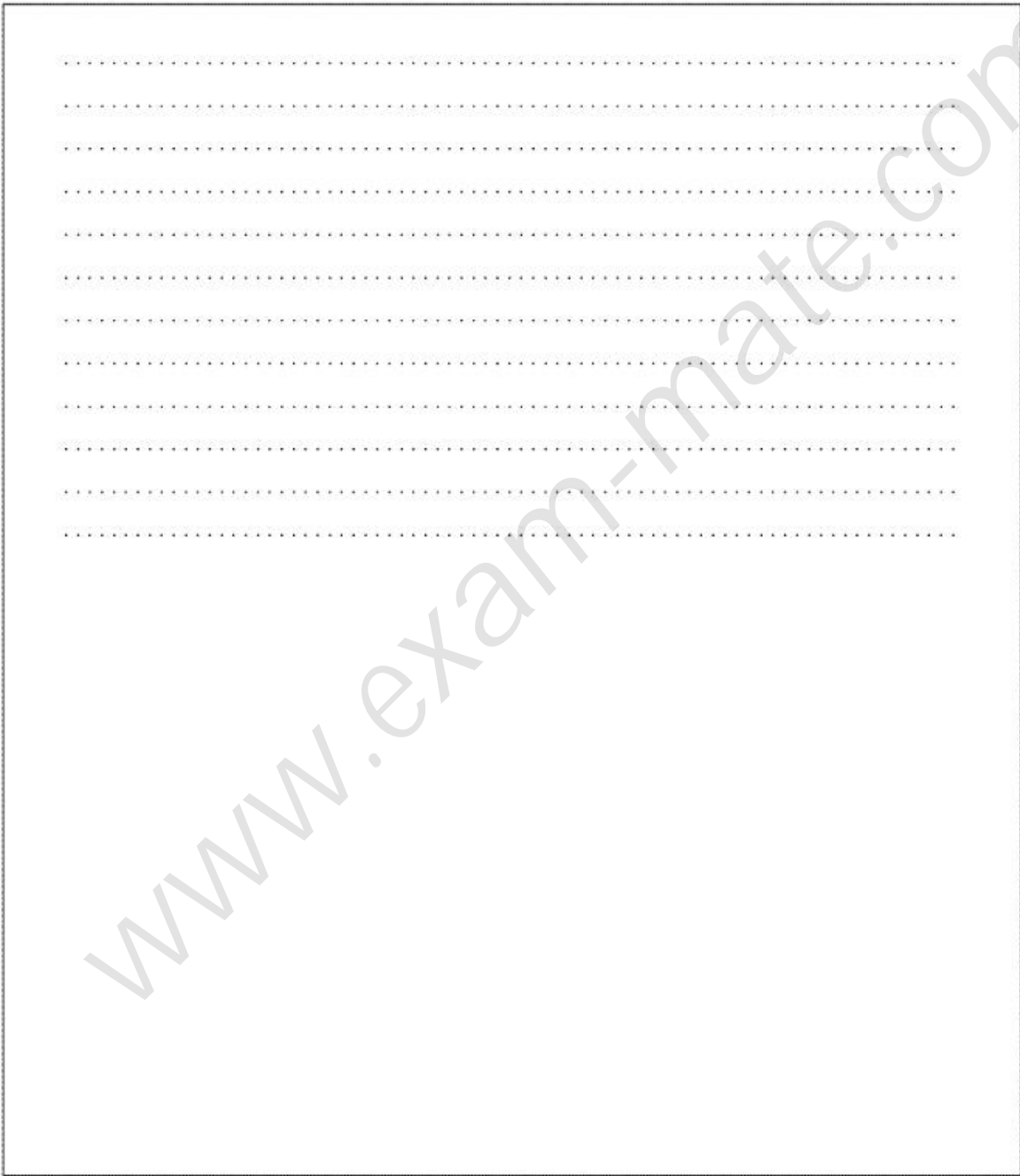
HL P2
2012 — 2023

Chapter 1	Functions - Roots	Page 1
Chapter 2	Quadratics	Page 17
Chapter 3	Exponentials - Logarithms	Page 21
Chapter 4	Graphs	Page 30
Chapter 5	Sequences - Series	Page 36
Chapter 6	Complex Numbers	Page 52
Chapter 7	Permutation - Combination	Page 63
Chapter 8	Binomial Theorem	Page 72
Chapter 9	Remainder & Factor Theorem	Page 78
Chapter 10	Mathematical Induction	Page 81
Chapter 11	Radian	Page 84
Chapter 12	Trigonometry	Page 94
Chapter 13	Matrices	Page 126
Chapter 14	Vectors - Lines - Planes	Page 132
Chapter 15	Statistics	Page 157
Chapter 16	Probability	Page 178
Chapter 17	Differentiation	Page 244
Chapter 18	Integration	Page 283
Chapter 19	Differential Equations	Page 309
Chapter 20	Kinematics	Page 315
Chapter 21	Geometry	-----
	Answers	Page 333

1 - (MAT/21_HL_Summer_2012_Q6) - Functions - Roots

[Maximum mark: 5]

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x -axis. Find an expression for $g(x)$, giving your answer as a single logarithm.



A large rectangular box containing horizontal dotted lines for writing the answer. A watermark 'www.exam-mate.com' is visible diagonally across the box.

2 - (MAT/22_HL_Summer_2014_Q7) - Functions - Roots, Graphs

The function f is defined as $f(x) = -3 + \frac{1}{x-2}$, $x \neq 2$.

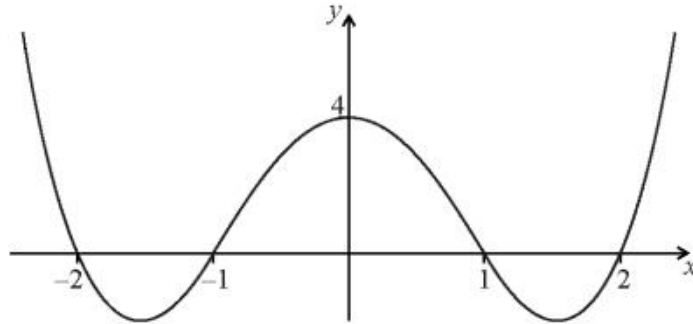
- (a) (i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.
(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts. [4]

- (b) Find the inverse function f^{-1} , stating its domain. [4]

3 - (MAT/21_HL_Summer_2014_Q12) - Functions - Roots, Graphs

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.



- (i) Find the value of $(f \circ g)(1)$.
- (ii) Find the value of $(f \circ g \circ g)(1)$.
- (iii) Sketch the graph of $y = (f \circ g)(x)$. [5]
- (b) (i) Sketch the graph of $y = f(x)$.
- (ii) State the zeros of f . [3]
- (c) (i) Sketch the graph of $y = (f \circ f)(x)$.
- (ii) State the zeros of $f \circ f$. [3]
- (d) Given that we can denote $\underbrace{f \circ f \circ f \circ \dots \circ f}_{n \text{ times}}$ as f^n ,
- (i) find the zeros of f^3 ;
- (ii) find the zeros of f^4 ;
- (iii) deduce the zeros of f^8 . [3]
- (e) The zeros of f^{2^n} are $a_1, a_2, a_3, \dots, a_N$.
- (i) State the relation between n and N ;
- (ii) Find, and simplify, an expression for $\sum_{r=1}^N |a_r|$ in terms of n . [4]

4 - (MAT/22_HL_Summer_2015_Q3) - *Graphs, Functions - Roots*

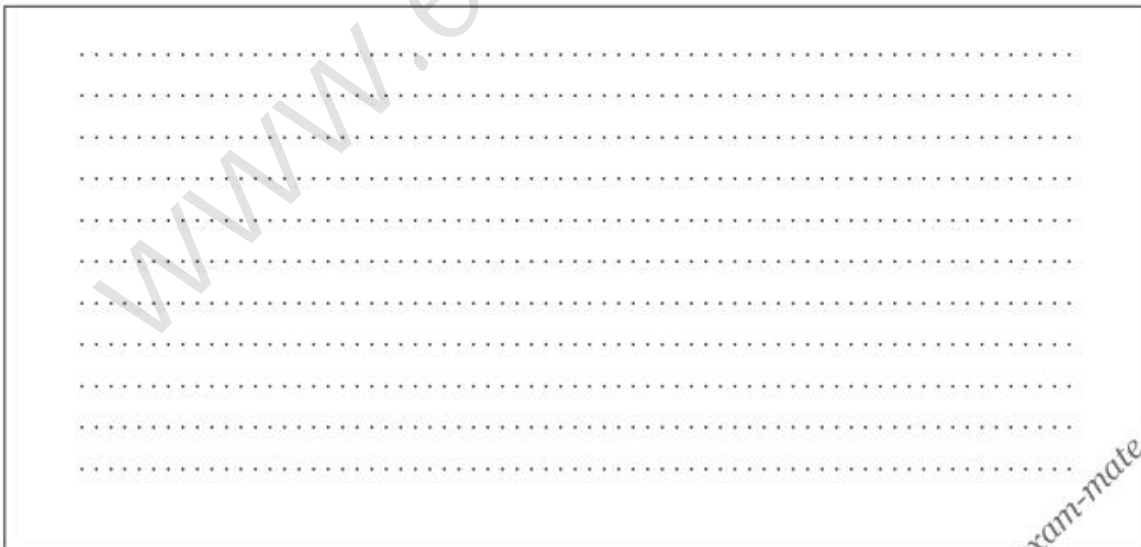
(a) Sketch the graph of $y = (x-5)^2 - 2|x-5| - 9$, for $0 \leq x \leq 10$.

[3]



(b) Hence, or otherwise, solve the equation $(x-5)^2 - 2|x-5| - 9 = 0$.

[2]



ANSWERS

www.examsmate.com

1 - (MAT/21_HL_Summer_2012_Q6) - Functions - Roots

$$h(x) = f(x-3) - 2 = \ln(x-3) - 2$$

$$g(x) = -h(x) = 2 - \ln(x-3)$$

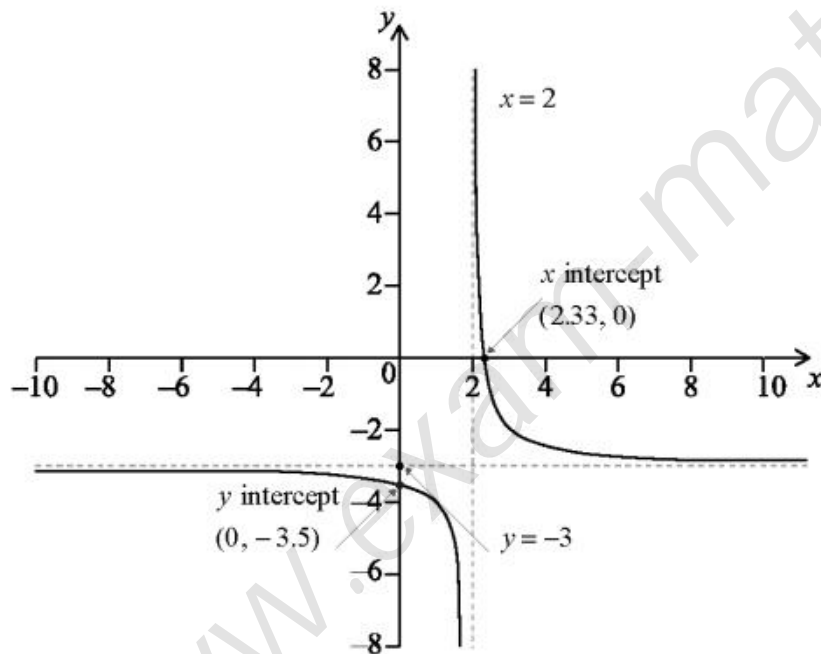
Note: Award M1 only if it is clear the effect of the reflection in the x -axis:
the expression is correct *OR*
there is a change of signs of the previous expression *OR*
there's a graph or an explanation making it explicit

$$= \ln e^2 - \ln(x-3)$$

$$= \ln\left(\frac{e^2}{x-3}\right)$$

2 - (MAT/22_HL_Summer_2014_Q7) - Functions - Roots, Graphs

(a)



AIAIAI

Note: Award *AI* for correct shape, *AI* for $x=2$ clearly stated and *AI* for $y=-3$ clearly stated.

x intercept (2.33, 0) and y intercept (0, -3.5)

AI

$$(b) \quad x = -3 + \frac{1}{y-2}$$

Note: Award *MI* for interchanging x and y (can be done at a later stage).

$$x+3 = \frac{1}{y-2}$$

$$y-2 = \frac{1}{x+3}$$

Note: Award *MI* for attempting to make y the subject.

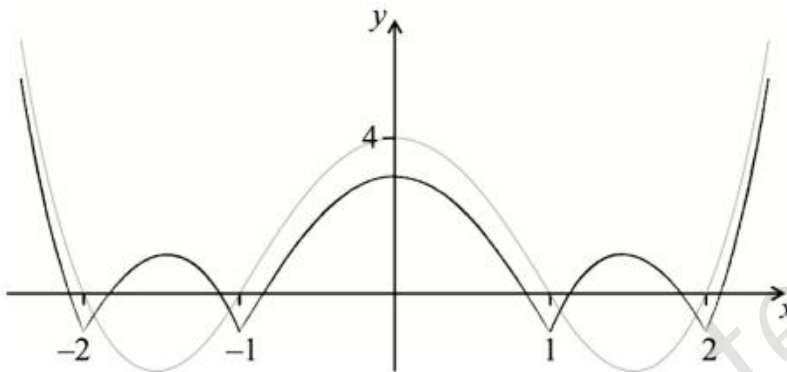
$$f^{-1}(x) = 2 + \frac{1}{x+3} \left(= \frac{2x+7}{x+3} \right), x \neq -3$$

3 - (MAT/21_HL_Summer_2014_Q12) - Functions - Roots, Graphs

(a) (i) $f(0) = -1$

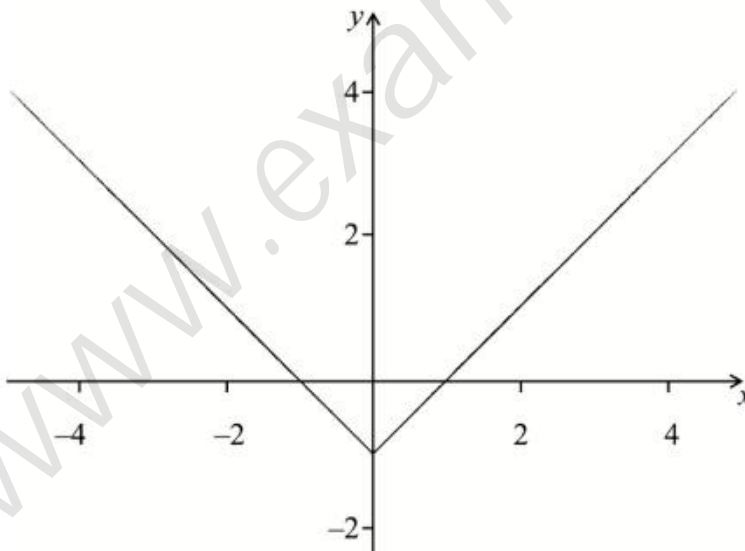
(ii) $(f \circ g)(0) = f(4) = 3$

(iii)



Note: Award *MI* for evidence that the lower part of the graph has been reflected and *AI* correct shape with *y*-intercept below 4.

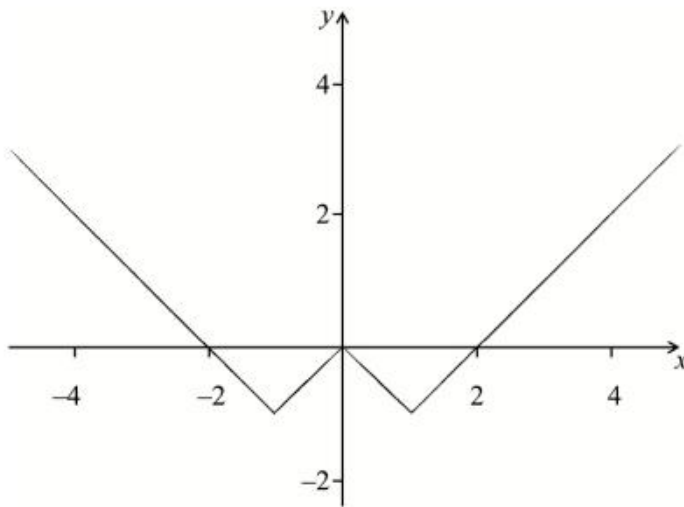
(b) (i)



Note: Award *MI* for any translation of $y = |x|$.

(ii) ± 1

(c) (i)



Note: Award *MI* for evidence that lower part of (b) has been reflected in the x -axis and translated.

(ii) $0, \pm 2$ (d) (i) $\pm 1, \pm 3$ (ii) $0, \pm 2, \pm 4$ (iii) $0, \pm 2, \pm 4, \pm 6, \pm 8$ (e) (i) $(1, 3), (2, 5), \dots$
 $N = 2n + 1$

(ii) Using the formula of the sum of an arithmetic series

EITHER

$$4(1 + 2 + 3 + \dots + n) = \frac{4}{2}n(n+1)$$

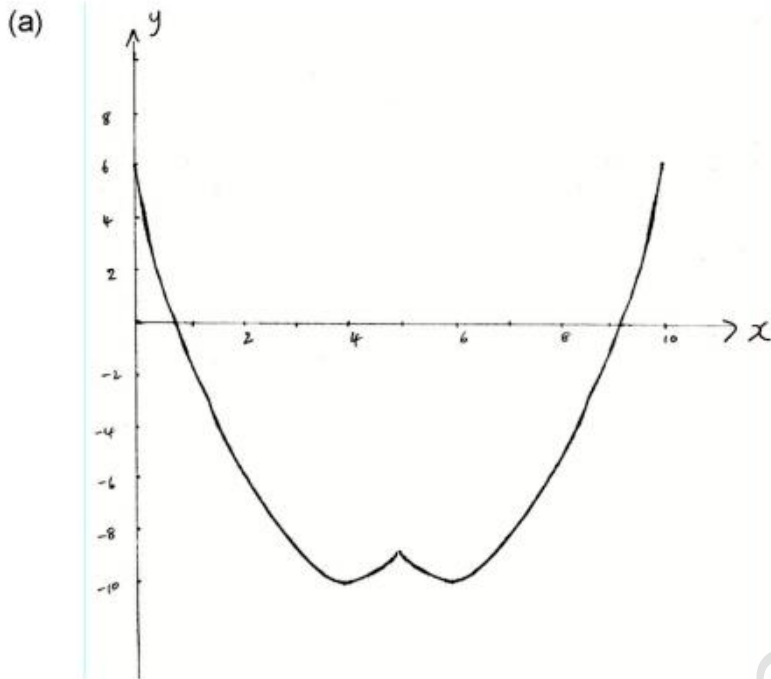
$$= 2n(n+1)$$

OR

$$2(2 + 4 + 6 + \dots + 2n) = \frac{2}{2}n(2n+2)$$

$$= 2n(n+1)$$

4 - (MAT/22_HL_Summer_2015_Q3) - Graphs, Functions - Roots



general shape including 2 minimums, cusp
correct domain and symmetrical about the middle ($x = 5$)

(b) $x = 9.16$ or $x = 0.838$