IB Diploma

MATHEMATICS AA
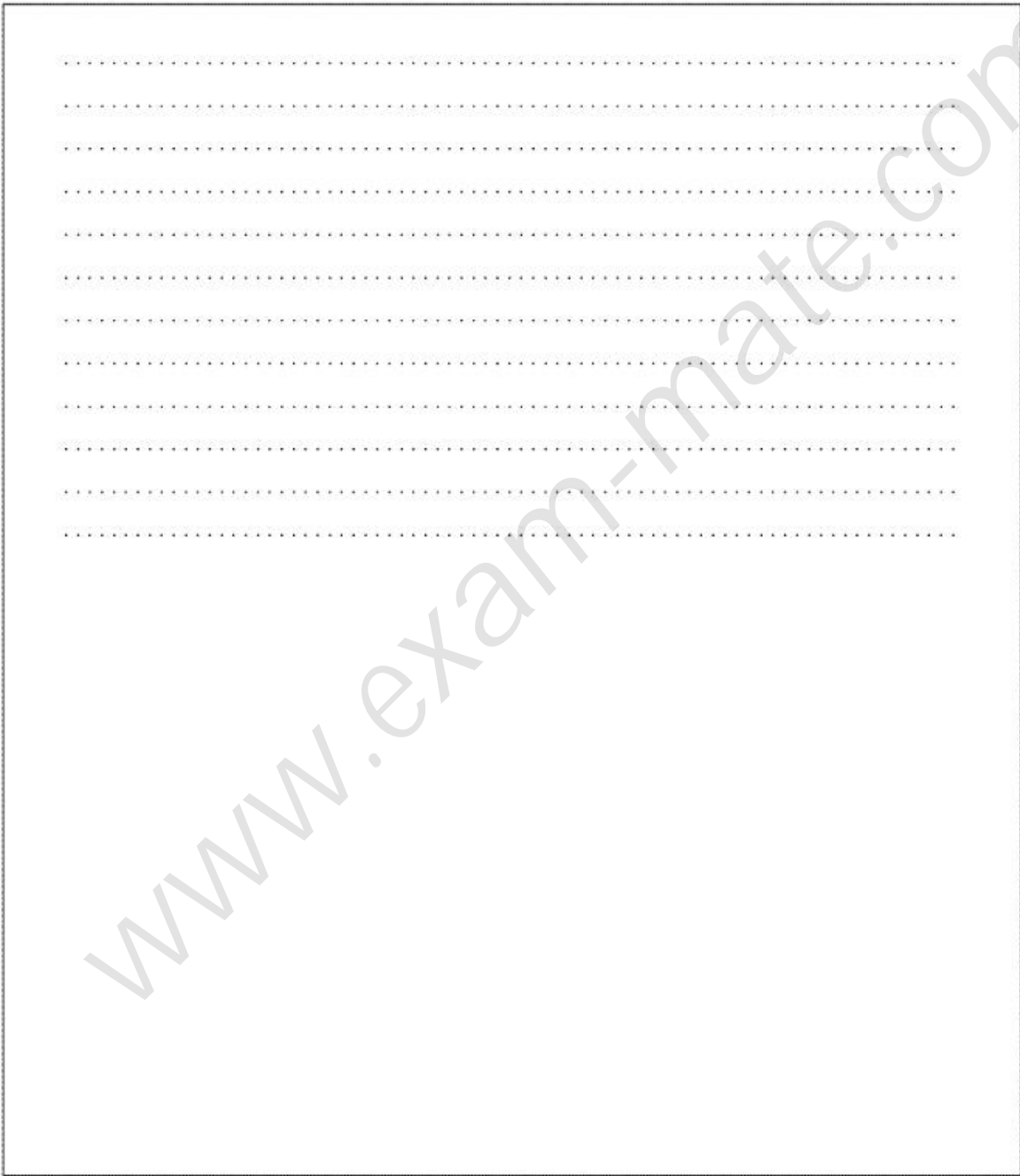
HL P2

2012 — 2023

1 - (MAT/21_HL_Summer_2012_Q6) - *Functions - Roots*

[Maximum mark: 5]

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g

by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression

for $g(x)$, giving your answer as a single logarithm.

2 - (MAT/22_HL_Summer_2014_Q7) - *Functions - Roots, Graphs*

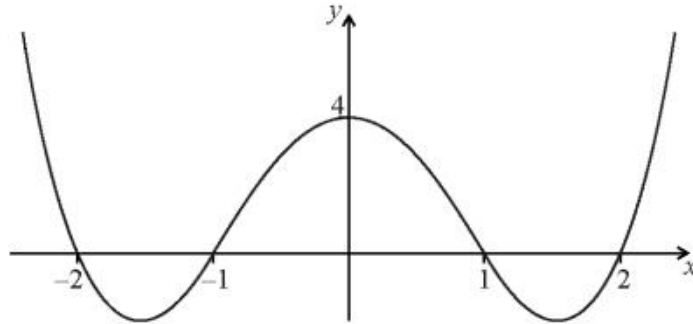The function f is defined as $f(x) = -3 + \dfrac{1}{x-2}$, $x \neq 2$.

(a) (i)   Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.

  (ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts.   [4]

(b) Find the inverse function f^{-1}, stating its domain.   [4]

3 – (MAT/21_HL_Summer_2014_Q12) – *Functions - Roots, Graphs*

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.

    (i) Find the value of $(f \circ g)(1)$.

    (ii) Find the value of $(f \circ g \circ g)(1)$.

    (iii) Sketch the graph of $y = (f \circ g)(x)$.   *[5]*

(b) (i) Sketch the graph of $y = f(x)$.

    (ii) State the zeros of f.   *[3]*

(c) (i) Sketch the graph of $y = (f \circ f)(x)$.

    (ii) State the zeros of $f \circ f$.   *[3]*

(d) Given that we can denote $\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}}$ as f^n,

    (i) find the zeros of f^3;

    (ii) find the zeros of f^4;

    (iii) deduce the zeros of f^8.   *[3]*

(e) The zeros of f^{2n} are $a_1, a_2, a_3, \ldots, a_N$.

    (i) State the relation between n and N;

    (ii) Find, and simplify, an expression for $\sum_{r=1}^{N} |a_r|$ in terms of n.   *[4]*

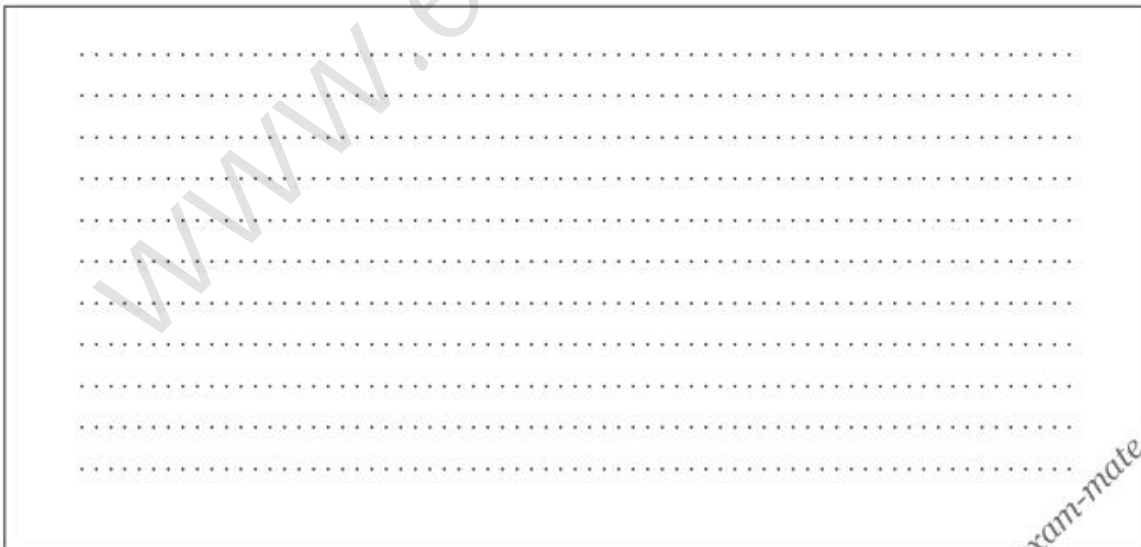**4** - (MAT/22_HL_Summer_2015_Q3) - *Graphs, Functions - Roots*

(a)   Sketch the graph of $y = (x-5)^2 - 2|x-5| - 9$, for $0 \le x \le 10$.   [3]

(b)   Hence, or otherwise, solve the equation $(x-5)^2 - 2|x-5| - 9 = 0$.   [2]

ANSWERS

1 ▪ (MAT/21_HL_Summer_2012_Q6) ▪ *Functions - Roots*

$$h(x) = f(x-3) - 2 = \ln(x-3) - 2$$
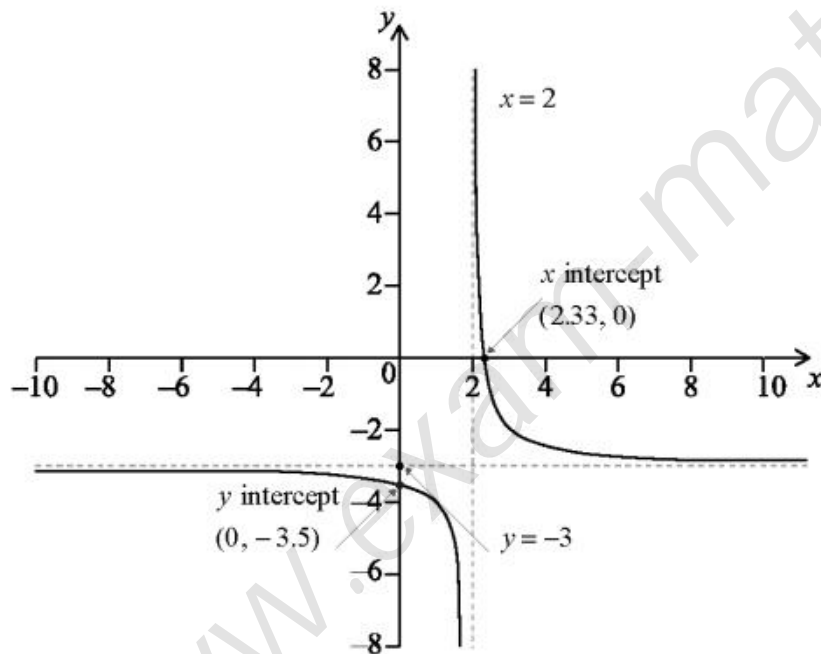$$g(x) = -h(x) = 2 - \ln(x-3)$$

> **Note:** Award M1 only if it is clear the effect of the reflection in the *x*-axis:
> the expression is correct *OR*
> there is a change of signs of the previous expression *OR*
> there's a graph or an explanation making it explicit

$$= \ln e^2 - \ln(x-3)$$
$$= \ln\left(\frac{e^2}{x-3}\right)$$

2 ▪ (MAT/22_HL_Summer_2014_Q7) ▪ *Functions - Roots, Graphs*

(a)

A1A1A1

> **Note:** Award *A1* for correct shape, *A1* for $x = 2$ clearly stated and *A1* for $y = -3$ clearly stated.

x intercept (2.33, 0) and *y* intercept (0, −3.5)   *A1*

(b) $x = -3 + \dfrac{1}{y-2}$

> **Note:**   Award *M1* for interchanging x and y (can be done at a later stage).

$$x + 3 = \dfrac{1}{y-2}$$

$$y - 2 = \dfrac{1}{x+3}$$

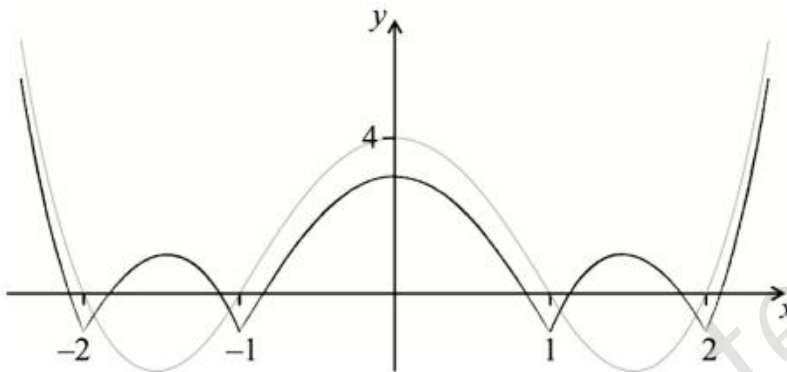> **Note:**   Award *M1* for attempting to make y the subject.

$$f^{-1}(x) = 2 + \dfrac{1}{x+3} \left(= \dfrac{2x+7}{x+3} \right), \; x \neq -3$$

3 - (MAT/21_HL_Summer_2014_Q12) - *Functions - Roots, Graphs*

(a)   (i) $f(0) = -1$
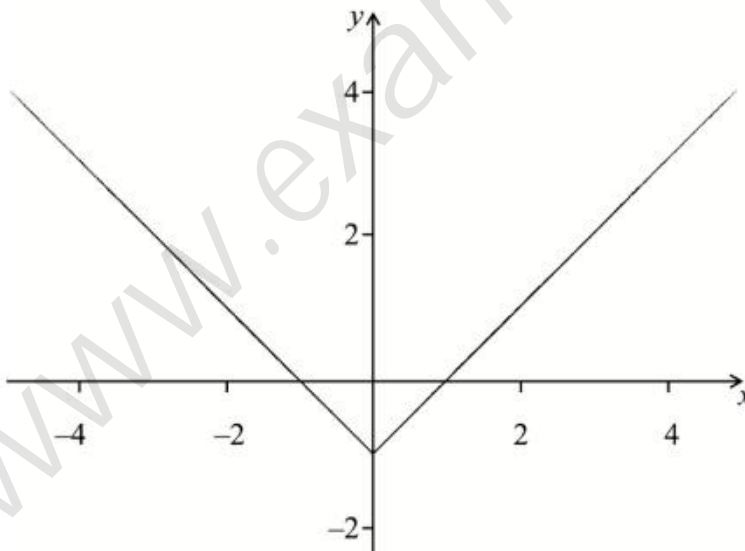
      (ii) $(f \circ g)(0) = f(4) = 3$

      (iii)

> **Note:**   Award *M1* for evidence that the lower part of the graph has been reflected and *A1* correct shape with y-intercept below 4.
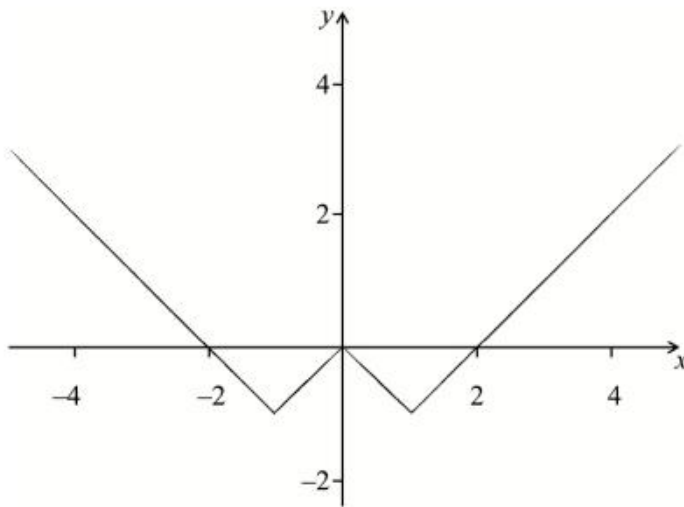
(b)   (i)

> **Note:**   Award *M1* for any translation of $y = |x|$.

      (ii) ± 1

(c) (i)

> **Note:** Award **M1** for evidence that lower part of (b) has been reflected in the x-axis and translated.

(ii) $0, \pm 2$

(d) (i) $\pm 1, \pm 3$

(ii) $0, \pm 2, \pm 4$

(iii) $0, \pm 2, \pm 4, \pm 6, \pm 8$

(e) (i) $(1, 3), (2, 5), \ldots$
$N = 2n + 1$

(ii) Using the formula of the sum of an arithmetic series
EITHER
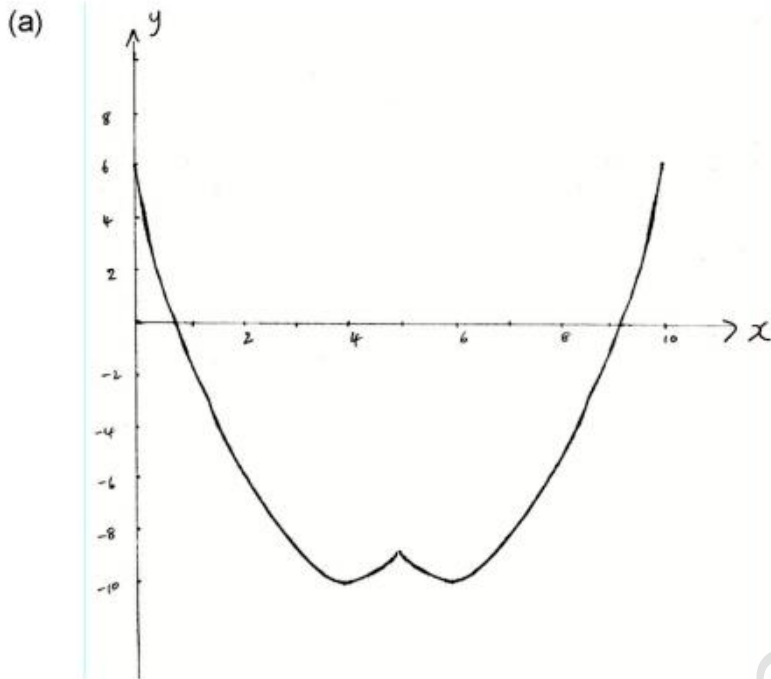
$$4(1 + 2 + 3 + \ldots + n) = \frac{4}{2}n(n+1)$$
$$= 2n(n+1)$$

OR

$$2(2 + 4 + 6 + \ldots + 2n) = \frac{2}{2}n(2n+2)$$
$$= 2n(n+1)$$

4 - (MAT/22_HL_Summer_2015_Q3) - *Graphs, Functions - Roots*

(a)

general shape including 2 minimums, cusp
correct domain and symmetrical about the middle $(x = 5)$

(b) $x = 9.16$ or $x = 0.838$