

IB Diploma

MATHEMATICS AA

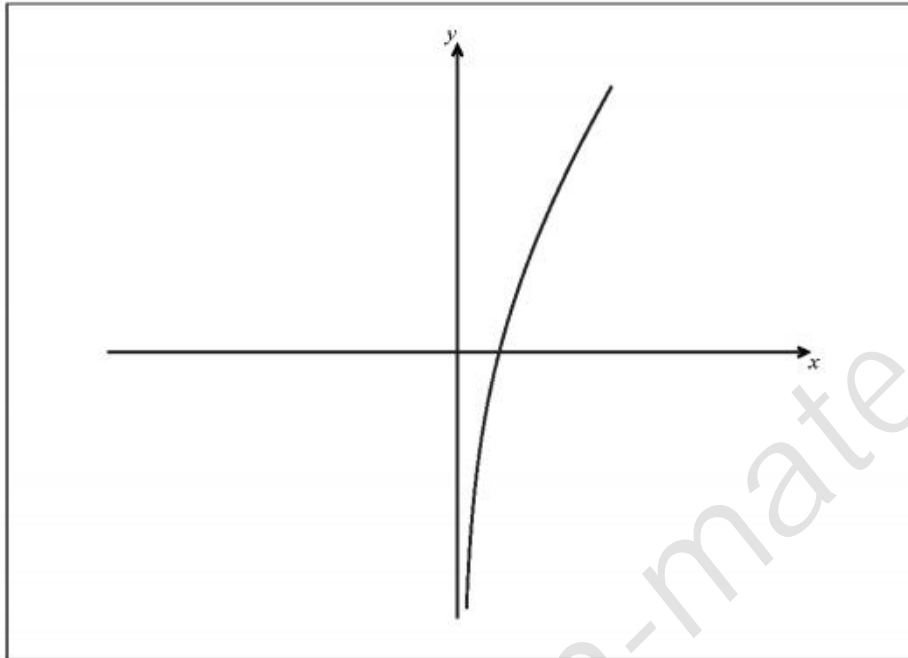
HL P1
2012 — 2023

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1 - (MAT/11_HL_Summer_2012_Q4) - Graphs, Functions - Roots

The graph below shows $y = f(x)$, where $f(x) = x + \ln x$.

- (a) On the graph below, sketch the curve $y = f^{-1}(x)$. [2 marks]



- (b) Find the coordinates of the point of intersection of the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. [4 marks]

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2 - (MAT/12_HL_Summer_2012_Q10) - Functions - Roots, Graphs, Integration

The function f is defined on the domain $\left[0, \frac{3\pi}{2}\right]$ by $f(x) = e^{-x} \cos x$.

(a) State the two zeros of f .

[1 mark]

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(b) Sketch the graph of f .

[1 mark]

- (c) The region bounded by the graph, the x -axis and the y -axis is denoted by A and the region bounded by the graph and the x -axis is denoted by B . Show that the ratio of the area of A to the area of B is

$$\frac{e^{\pi} \left(e^{\frac{\pi}{2}} + 1 \right)}{e^{\pi} + 1}.$$

[7 marks]

A large rectangular area with horizontal dotted lines for writing the solution. A diagonal watermark 'www.exam-mate.com' is visible across the area.

3 - (MAT/12_HL_Summer_2012_Q11) - Functions - Roots, Probability

Consider the following functions:

$$f(x) = \frac{2x^2 + 3}{75}, x \geq 0$$

$$g(x) = \frac{|3x - 4|}{10}, x \in \mathbb{R}.$$

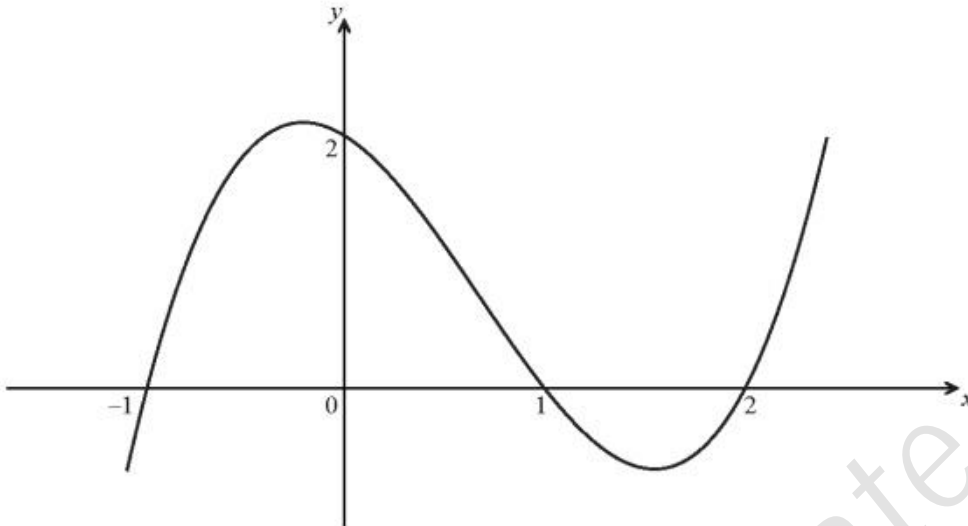
- (a) State the range of f and of g . [2 marks]
- (b) Find an expression for the composite function $f \circ g(x)$ in the form $\frac{ax^2 + bx + c}{3750}$, where a, b and $c \in \mathbb{Z}$. [4 marks]
- (c) (i) Find an expression for the inverse function $f^{-1}(x)$.
- (ii) State the domain and range of f^{-1} . [4 marks]

The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$.

- (d) By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X , stating your reasons clearly. [6 marks]
- (e) Using this probability distribution, calculate the mean of X . [2 marks]

4 - (MAT/10_HL_Winter_2012_Q3) - Functions - Roots, Graphs

Let $f(x) = x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{Z}$. The diagram shows the graph of $y = f(x)$.



- (a) Using the information shown in the diagram, find the values of a , b and c . [4 marks]

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(b) If $g(x) = 3f(x-2)$,

- (i) state the coordinates of the points where the graph of g intercepts the x -axis.

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- (ii) Find the y -intercept of the graph of g .

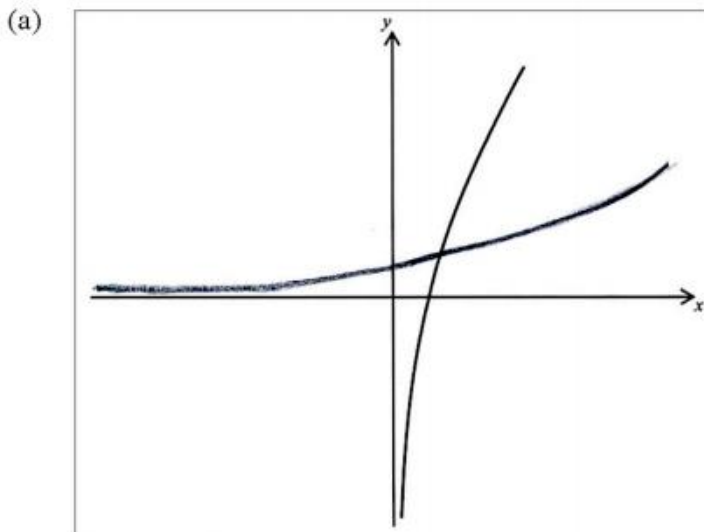
[3 marks]

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ANSWERS

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1 - (MAT/11_HL_Summer_2012_Q4) - Graphs, Functions - Roots

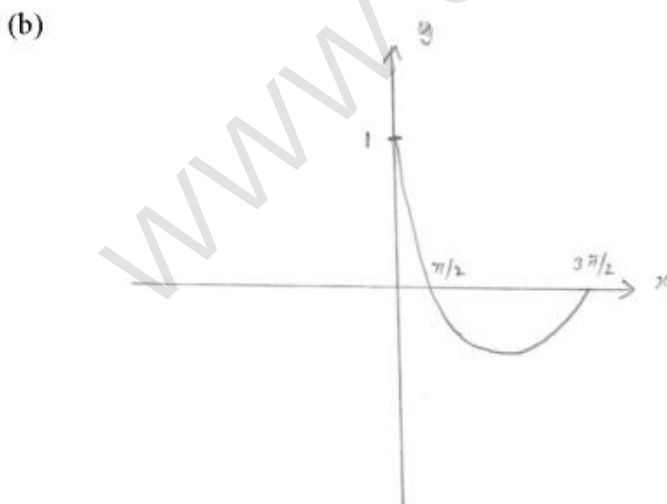


Note: Award *AI* for correct asymptote with correct behaviour and *AI* for shape.

- (b) intersect on $y = x$
 $x + \ln x = x \Rightarrow \ln x = 0$
 intersect at $(1, 1)$

2 - (MAT/12_HL_Summer_2012_Q10) - Functions - Roots, Graphs, Integration

(a) $e^{-x} \cos x = 0$
 $\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$



(c) attempt at integration by parts

EITHER

$$\begin{aligned} I &= \int e^{-x} \cos x dx = -e^{-x} \cos x - \int e^{-x} \sin x dx \\ \Rightarrow I &= -e^{-x} \cos x - \left[-e^{-x} \sin x + \int e^{-x} \cos x dx \right] \\ \Rightarrow I &= \frac{e^{-x}}{2} (\sin x - \cos x) + C \end{aligned}$$

Note: Do not penalize absence of C .

OR

$$\begin{aligned} I &= \int e^{-x} \cos x dx = e^{-x} \sin x + \int e^{-x} \sin x dx \\ \Rightarrow I &= e^{-x} \sin x - e^{-x} \cos x - \int e^{-x} \cos x dx \\ \Rightarrow I &= \frac{e^{-x}}{2} (\sin x - \cos x) + C \end{aligned}$$

Note: Do not penalize absence of C .

THEN

$$\int_0^{\frac{\pi}{2}} e^{-x} \cos x dx = \left[\frac{e^{-x}}{2} (\sin x - \cos x) \right]_0^{\frac{\pi}{2}} = \frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{-x} \cos x dx = \left[\frac{e^{-x}}{2} (\sin x - \cos x) \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -\frac{e^{-\frac{3\pi}{2}}}{2} - \frac{e^{-\frac{\pi}{2}}}{2}$$

ratio of $A:B$ is $\frac{\frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}}{\frac{e^{-\frac{3\pi}{2}}}{2} + \frac{e^{-\frac{\pi}{2}}}{2}}$

$$\begin{aligned} &= \frac{e^{\frac{3\pi}{2}} \left(e^{\frac{\pi}{2}} + 1 \right)}{e^{\frac{3\pi}{2}} \left(e^{\frac{3\pi}{2}} + e^{\frac{\pi}{2}} \right)} \\ &= \frac{e^{\pi} \left(e^{\frac{\pi}{2}} + 1 \right)}{e^{\pi} + 1} \end{aligned}$$

3 - (MAT/12_HL_Summer_2012_Q11) - Functions - Roots, Probability

(a) $f(x) \geq \frac{1}{25}$
 $g(x) \in \mathbb{R}, g(x) \geq 0$

(b) $f \circ g(x) = \frac{2\left(\frac{3x-4}{10}\right)^2 + 3}{75}$
 $= \frac{2(9x^2 - 24x + 16)}{100} + 3$
 $= \frac{9x^2 - 24x + 166}{3750}$

(c) (i) METHOD 1

$$y = \frac{2x^2 + 3}{75}$$

$$x^2 = \frac{75y - 3}{2}$$

$$x = \sqrt{\frac{75y - 3}{2}}$$

$$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x - 3}{2}}$$

Note: Accept \pm in line 3 for the *(AI)* but not in line 4 for the *AI*.
 Award the *AI* only if written in the form $f^{-1}(x) =$.

METHOD 2

$$y = \frac{2x^2 + 3}{75}$$

$$x = \frac{2y^2 + 3}{75}$$

$$y = \sqrt{\frac{75x - 3}{2}}$$

$$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x - 3}{2}}$$

Note: Accept \pm in line 3 for the *(AI)* but not in line 4 for the *AI*.
 Award the *AI* only if written in the form $f^{-1}(x) =$.

(ii) domain: $x \geq \frac{1}{25}$; range: $f^{-1}(x) \geq 0$

(d) probabilities from $f(x)$:

X	0	1	2	3	4
$P(X = x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

Note: Award **A1** for one error, **A0** otherwise.probabilities from $g(x)$:

X	0	1	2	3	4
$P(X = x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

Note: Award **A1** for one error, **A0** otherwise.

only in the case of $f(x)$ does $\sum P(X = x) = 1$, hence only $f(x)$ can be used as a probability mass function

(e)

$$E(x) = \sum x \cdot P(X = x)$$

$$= \frac{5}{75} + \frac{22}{75} + \frac{63}{75} + \frac{140}{75} = \frac{230}{75} \left(= \frac{46}{15} \right)$$

4 - (MAT/10_HL_Winter_2012_Q3) - Functions - Roots, Graphs

(a) **METHOD 1**

$$f(x) = (x+1)(x-1)(x-2)$$

$$= x^3 - 2x^2 - x + 2$$

$$a = -2, b = -1 \text{ and } c = 2$$

METHOD 2from the graph or using $f(0) = 2$

$$c = 2$$

setting up linear equations using $f(1) = 0$ and $f(-1) = 0$ (or $f(2) = 0$)obtain $a = -2, b = -1$

(b) (i) (1, 0), (3, 0) and (4, 0)

(ii) $g(0)$ occurs at $3f(-2)$

$$= -36$$