IB Diploma

MATHEMATICS AA

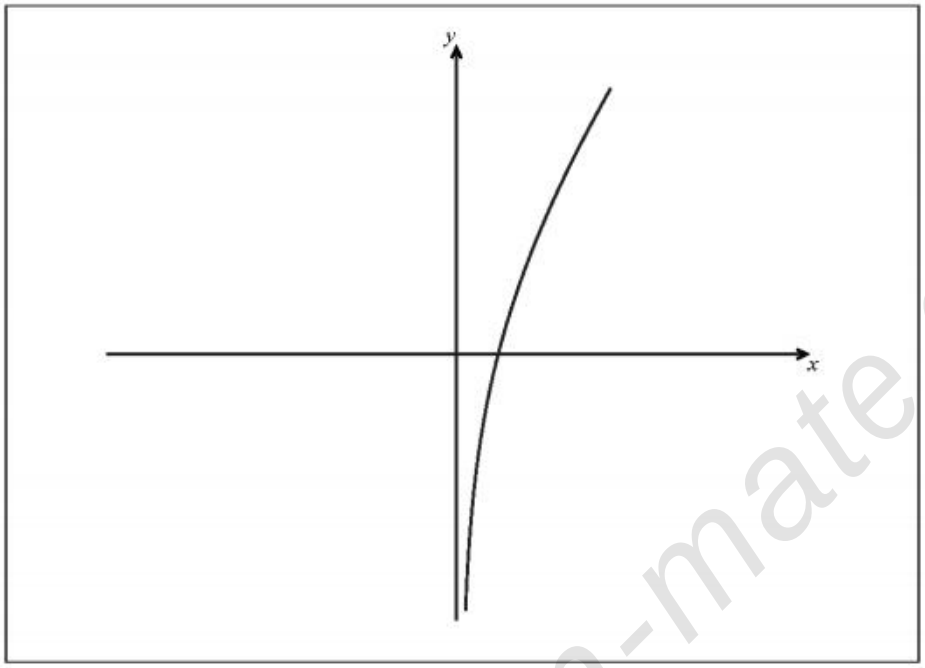
HL P1

2012 — 2023

1 – (MAT/11_HL_Summer_2012_Q4) – *Graphs, Functions - Roots*

The graph below shows $y = f(x)$, where $f(x) = x + \ln x$.

(a) On the graph below, sketch the curve $y = f^{-1}(x)$. *[2 marks]*



(b) Find the coordinates of the point of intersection of the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. *[4 marks]*

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2 – (MAT/12_HL_Summer_2012_Q10) – *Functions - Roots, Graphs, Integration*

The function f is defined on the domain $\left[0, \frac{3\pi}{2}\right]$ by $f(x) = e^{-x}\cos x$.

(a) State the two zeros of f. *[1 mark]*

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(b) Sketch the graph of f. *[1 mark]*

(c) The region bounded by the graph, the x-axis and the y-axis is denoted by A and the region bounded by the graph and the x-axis is denoted by B. Show that the ratio of the area of A to the area of B is

$$\frac{e^{\pi}\left(e^{\frac{\pi}{2}}+1\right)}{e^{\pi}+1}.$$

[7 marks]

3 – (MAT/12_HL_Summer_2012_Q11) – *Functions - Roots, Probability*

Consider the following functions:

$$f(x)=\frac{2x^2+3}{75},\ x\geq 0$$

$$g(x)=\frac{|3x-4|}{10},\ x\in\mathbb{R}.$$

(a) State the range of f and of g. *[2 marks]*

(b) Find an expression for the composite function $f\circ g(x)$ in the form $\frac{ax^2+bx+c}{3750}$, where a, b and $c\in\mathbb{Z}$. *[4 marks]*

(c) (i) Find an expression for the inverse function $f^{-1}(x)$.

(ii) State the domain and range of f^{-1}. *[4 marks]*

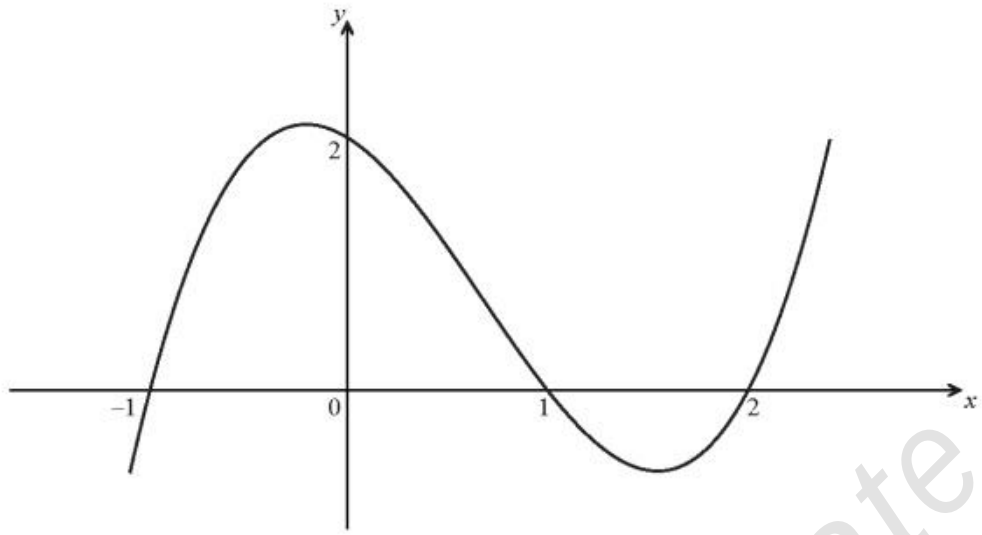
The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$.

(d) By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X, stating your reasons clearly. *[6 marks]*

(e) Using this probability distribution, calculate the mean of X. *[2 marks]*

4 – (MAT/10_HL_Winter_2012_Q3) – *Functions - Roots, Graphs*

Let $f(x)=x^3+ax^2+bx+c$, where $a,\ b,\ c\in\mathbb{Z}$. The diagram shows the graph of $y=f(x)$.



(a) Using the information shown in the diagram, find the values of a, b and c. *[4 marks]*

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(b) If $g(x)=3f(x-2)$,

(i) state the coordinates of the points where the graph of g intercepts the x-axis.

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(ii) Find the y-intercept of the graph of g. *[3 marks]*

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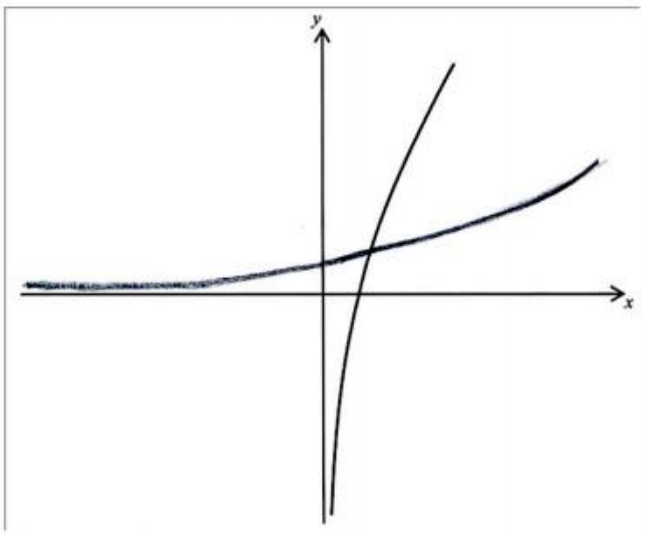
ANSWERS

1 – (MAT/11_HL_Summer_2012_Q4) – *Graphs, Functions - Roots*

(a)



Note: Award *A1* for correct asymptote with correct behaviour and *A1* for shape.

(b) intersect on $y = x$

$x + \ln x = x \Rightarrow \ln x = 0$

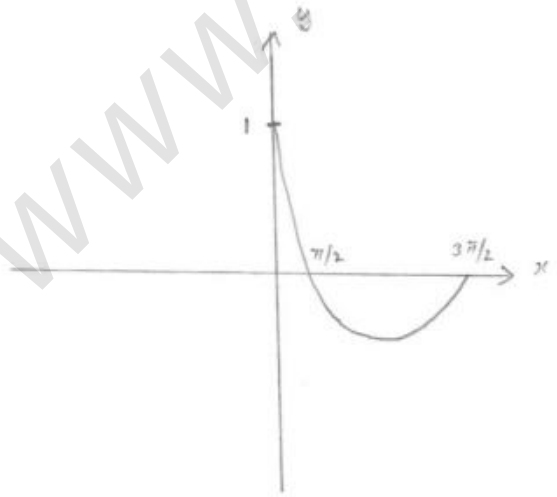
intersect at $(1, 1)$

2 – (MAT/12_HL_Summer_2012_Q10) – *Functions - Roots, Graphs, Integration*

(a) $e^{-x}\cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

(b)



(c) attempt at integration by parts

EITHER

$$I=\int e^{-x}\cos x\,dx=-e^{-x}\cos x\,dx-\int e^{-x}\sin x\,dx$$

$$\Rightarrow I=-e^{-x}\cos x\,dx-\left[-e^{-x}\sin x+\int e^{-x}\cos x\,dx\right]$$

$$\Rightarrow I=\frac{e^{-x}}{2}(\sin x-\cos x)+C$$

Note: Do not penalize absence of C.

OR

$$I=\int e^{-x}\cos x\,dx=e^{-x}\sin x+\int e^{-x}\sin x\,dx$$

$$\Rightarrow I=e^{-x}\sin x-e^{-x}\cos x-\int e^{-x}\cos x\,dx$$

$$\Rightarrow I=\frac{e^{-x}}{2}(\sin x-\cos x)+C$$

Note: Do not penalize absence of C.

THEN

$$\int_0^{\frac{\pi}{2}}e^{-x}\cos x\,dx=\left[\frac{e^{-x}}{2}(\sin x-\cos x)\right]_0^{\frac{\pi}{2}}=\frac{e^{-\frac{\pi}{2}}}{2}+\frac{1}{2}$$

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}e^{-x}\cos x\,dx=\left[\frac{e^{-x}}{2}(\sin x-\cos x)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}=-\frac{e^{-\frac{3\pi}{2}}}{2}-\frac{e^{-\frac{\pi}{2}}}{2}$$

ratio of $A:B$ is $\dfrac{\frac{e^{-\frac{\pi}{2}}}{2}+\frac{1}{2}}{\frac{e^{-\frac{3\pi}{2}}}{2}+\frac{e^{-\frac{\pi}{2}}}{2}}$

$$=\frac{e^{\frac{3\pi}{2}}\left(e^{-\frac{\pi}{2}}+1\right)}{e^{\frac{3\pi}{2}}\left(e^{-\frac{3\pi}{2}}+e^{-\frac{\pi}{2}}\right)}$$

$$=\frac{e^{\pi}\left(e^{\frac{\pi}{2}}+1\right)}{e^{\pi}+1}$$

3 – (MAT/12_HL_Summer_2012_Q11) – *Functions - Roots, Probability*

(a) $f(x) \geq \frac{1}{25}$

$g(x) \in \mathbb{R}, g(x) \geq 0$

(b) $f \circ g(x) = \frac{2\left(\frac{3x-4}{10}\right)^2 + 3}{75}$

$= \frac{\frac{2(9x^2-24x+16)}{100} + 3}{75}$

$= \frac{9x^2-24x+166}{3750}$

(c) (i) **METHOD 1**

$y = \frac{2x^2+3}{75}$

$x^2 = \frac{75y-3}{2}$

$x = \sqrt{\frac{75y-3}{2}}$

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$

Note:	Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***. Award the ***A1*** only if written in the form $f^{-1}(x) =$.

METHOD 2

$y = \frac{2x^2+3}{75}$

$x = \frac{2y^2+3}{75}$

$y = \sqrt{\frac{75x-3}{2}}$

$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}}$

Note:	Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***. Award the ***A1*** only if written in the form $f^{-1}(x) =$.

(ii) domain: $x \geq \frac{1}{25}$; range: $f^{-1}(x) \geq 0$

(d) probabilities from $f(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

Note: Award ***A1*** for one error, ***A0*** otherwise.

probabilities from $g(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

Note: Award ***A1*** for one error, ***A0*** otherwise.

only in the case of $f(x)$ does $\sum P(X=x)=1$, hence only $f(x)$ can be used as a probability mass function

(e)

$$E(x)=\sum x\cdot \mathrm{P}(X=x)$$

$$=\frac{5}{75}+\frac{22}{75}+\frac{63}{75}+\frac{140}{75}=\frac{230}{75}\left(=\frac{46}{15}\right)$$

4 – (MAT/10_HL_Winter_2012_Q3) – *Functions - Roots, Graphs*

(a) **METHOD 1**

$f(x)=(x+1)(x-1)(x-2)$

$=x^3-2x^2-x+2$

$a=-2$, $b=-1$ and $c=2$

METHOD 2

from the graph or using $f(0)=2$

$c=2$

setting up linear equations using $f(1)=0$ and $f(-1)=0$ (or $f(2)=0$)

obtain $a=-2$, $b=-1$

(b) (i) $(1, 0)$, $(3, 0)$ and $(4, 0)$

(ii) $g(0)$ occurs at $3f(-2)$

$=-36$