IB Diploma

# MATHEMATICS AA

## SL P2

2012 — 2023

**1** – (MAT/21_SL_Summer_2013_Q6) – *Functions - Roots, Graphs*

Let $f$ and $g$ be functions such that $g(x) = 2f(x+1)+5$.

(a) The graph of $f$ is mapped to the graph of $g$ under the following transformations:

vertical stretch by a factor of $k$, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) $k$;

(ii) $p$;

(iii) $q$. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point A$(6, 5)$ on the graph of $g$ is mapped to the point A′ on the graph of $h$. Find A′. *[3 marks]*

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**2** – (MAT/20_SL_Winter_2014_Q1) – *Functions - Roots*

Let $f(x)=2x+3$ and $g(x)=x^3$.

(a) Find $(f\circ g)(x)$. *[2]*

(b) Solve the equation $(f\circ g)(x)=0$. *[3]*

**3** – (MAT/21_SL_Summer_2015_Q4) – *Functions - Roots*

Let $f(x)=\dfrac{2x-6}{1-x}$, for $x \neq 1$.

(a) For the graph of $f$

(i) find the $x$-intercept;

(ii) write down the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote. [5]

(b) Find $\lim_{x\to\infty} f(x)$. [2]

**4** – (MAT/20_SL_Winter_2016_Q1) – *Functions - Roots*

Let $f(x)=x^2+2x+1$ and $g(x)=x-5$, for $x\in\mathbb{R}$.

(a) Find $f(8)$. [2]

(b) Find $(g\circ f)(x)$. [2]

(c) Solve $(g\circ f)(x)=0$. [3]

**5** – (MAT/21_SL_Summer_2014_Q10) – *Functions - Roots, Differentiation*

Let $f(x)=\dfrac{3x}{x-q}$, where $x\neq q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of $f$. *[2]*

The vertical and horizontal asymptotes to the graph of $f$ intersect at the point $Q(1, 3)$.

(b) Find the value of $q$. *[2]*

(c) The point $P(x, y)$ lies on the graph of $f$. Show that $PQ=\sqrt{(x-1)^2+\left(\dfrac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of $f$ that are closest to $(1, 3)$. *[6]*

# ANSWERS

**1** – (MAT/21_SL_Summer_2013_Q6) – *Functions - Roots, Graphs*

(a) (i) $k = 2$

(ii) $p = -1$

(iii) $q = 5$

(b) recognizing one transformation

*eg* horizontal stretch by $\frac{1}{3}$, reflection in $x$-axis,

A′ is $(2, -5)$

**2** – (MAT/20_SL_Winter_2014_Q1) – *Functions - Roots*

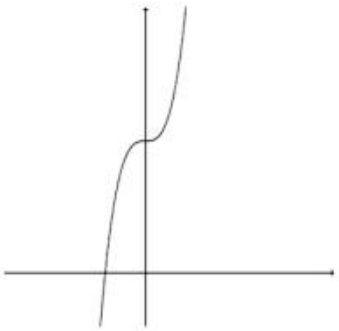
(a) attempt to form composite (in any order)

*eg* $f\left(x^3\right),\ (2x+3)^3$

$(f\circ g)(x)=2x^3+3,\ 2\left(x\right)^3+3$

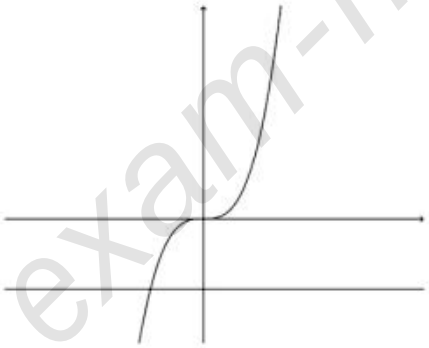
(b) evidence of appropriate approach

*eg* $2x^3=-3$, sketch



correct working

*eg* $x^3=\frac{-3}{2}$, sketch



$-1.14471$

$x=\sqrt[3]{\frac{-3}{2}}$ (exact), $-1.14$ $[-1.15, -1.14]$

**3** – (MAT/21_SL_Summer_2015_Q4) – *Functions - Roots*

(a) (i) valid approach
*eg* sketch, $f(x)=0$, $0=2x-6$

$x=3$ or $(3, 0)$

(ii) $x=1$ (must be equation)

(iii) valid approach
*eg* sketch, $\frac{2x}{-1x}$, inputting large values of $x$, L'Hopital's rule

$y=-2$ (must be equation)

(b) valid approach
*eg* recognizing that $\lim_{x\to\infty}$ is related to the horizontal asymptote,
table with large values of $x$, their $y$ value from (a)(iii), L'Hopital's rule

$\lim_{x\to\infty} f(x)=-2$

**4** – (MAT/20_SL_Winter_2016_Q1) – *Functions - Roots*

(a) attempt to substitute $x=8$ ***(M1)***
*eg* $8^2+2\times 8+1$

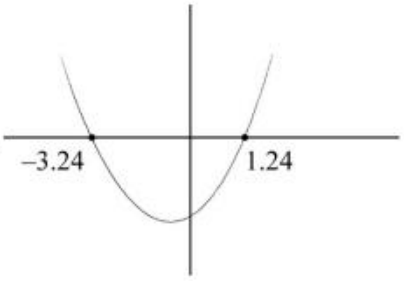
$f(8)=81$ ***A1*** ***N2***
***[2 marks]***

(b) attempt to form composition (in any order) ***(M1)***
*eg* $f(x-5)$, $g(f(x))$, $(x^2+2x+1)-5$

$(g\circ f)(x)=x^2+2x-4$ ***A1*** ***N2***
***[2 marks]***

(c) valid approach ***(M1)***

*eg* $x=\frac{-2\pm\sqrt{20}}{2}$,




$1.23606, -3.23606$

$x=1.24$, $x=-3.24$ ***A1A1*** ***N3***
***[3 marks]***

**5** – (MAT/21_SL_Summer_2014_Q10) – *Functions - Roots, Differentiation*

(a) $x = q,\ y = 3$ (must be equations)

(b) recognizing connection between point of intersection and asymptote
*eg* $x = 1$

$q = 1$

(c) correct substitution into distance formula
*eg* $\sqrt{(x-1)^2 + (y-3)^2}$

attempt to substitute $y = \frac{3x}{x-1}$
*eg* $\sqrt{(x-1)^2 + \left(\frac{3x}{x-1} - 3\right)^2}$

correct simplification of $\left(\frac{3x}{x-1} - 3\right)$
*eg* $\frac{3x - 3(x-1)}{x-1}$

correct expression clearly leading to the required answer
*eg* $\frac{3x - 3x + 3}{x-1},\ \sqrt{(x-1)^2 + \left(\frac{3x-3x+3}{x-1}\right)^2}$

$\text{PQ} = \sqrt{(x-1)^2 + \left(\frac{3}{x-1}\right)^2}$

(d) recognizing that closest is when PQ is a minimum
*eg* sketch of PQ, $(\text{PQ})'(x) = 0$

$x = -0.73205\ x = 2.73205$ (seen anywhere)

attempt to find $y$-coordinates
*eg* $f(-0.73205)$

$(-0.73205, 1.267949),\ (2.73205, 4.73205)$
$(-0.732, 1.27),\ (2.73, 4.73)$