A LEVEL Cambridge Topical Past Papers

FURTHER STATISTICS

2017— 2023

FURTHER MATHEMATICS P4 9231

TOPICAL PAST PAPER WORKSHEETS

2017 - 2023 | Questions + Mark scheme

AVAILABLE PAPERS

P1,2	P3	P4
395 Questions	179 Questions	164 Questions

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1 - (9231/21_Summer_2017_Q6) - *Further Work On Distributions*

A fair die is thrown repeatedly until a 6 is obtained.

(i) Find the probability that obtaining a 6 takes no more than four throws. [2]

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(ii) Find the least integer N such that the probability of obtaining a 6 before the Nth throw is more than 0.95. [3]

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2 - (9231/21_Summer_2017_Q8) - *Further Work On Distributions*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{4}(x-1) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

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The random variable Y is defined by $Y = (X-1)^3$.

(ii) Find the probability density function of Y. [4]

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(iii) Find the median value of Y. [3]

3 - (9231/23_Summer_2017_Q9) - *Further Work On Distributions*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 0 & x < 0, \\ ae^{-x\ln 2} & x \geqslant 0, \end{cases}$$

where a is a positive constant.

(i) Find the value of a. [2]

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(ii) State the value of $E(X)$. [1]

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(iii) Find the interquartile range of X. [4]

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The variable Y is related to X by $Y = 2^X$.

(iv) Find the probability density function of Y. [5]

4 - (9231/21_Winter_2017_Q6) - *Further Work On Distributions*

A pair of fair dice is thrown repeatedly until a pair of sixes is obtained. The number of throws taken is denoted by the random variable X.

(i) Find the mean value of X. [2]

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(ii) Find the probability that exactly 12 throws are required to obtain a pair of sixes. [2]

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(iii) Find the probability that more than 12 throws are required to obtain a pair of sixes. [2]

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5 - (9231/21_Winter_2017_Q7) - *Further Work On Distributions*

The random variable X has probability density function f given by

$$f(x) = \begin{cases} 0.2e^{-0.2x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [2]

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(ii) Find $P(X > 2)$. [2]

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(iii) Find the median of X. [3]

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ANSWERS

1 - (9231/21_Summer_2017_Q6) - *Further Work On Distributions*

(i)	$P(X \le 4) = 1 - q^4$	**M1**
	$= 671/1296$ *or* $0{\cdot}518$	**A1**
	Total:	**2**
(ii)	$1 - q^{N-1} > 0{\cdot}95$	**M1**
	$(5/6)^{N-1} < 0{\cdot}05$, $N - 1 > \log 0{\cdot}05 / \log 5/6$	**M1**
	$N - 1 > 16{\cdot}4[3]$, $N_{\min} = 18$	**A1**
	Total:	**3**

2 - (9231/21_Summer_2017_Q8) - *Further Work On Distributions*

(i)	$F(x) = \int f(x)\,dx = x^2/8 - x/4\ [+ c]$	**M1**
	$= x^2/8 - x/4$ *or* $\{(x-1)^2 - 1\}/8$ (AEF)	**A1**
	$F(x) = 0\ (x < 2)$, $F(x) = 1\ (x > 4)$	**A1**
	Total:	**3**
(ii)	*EITHER:* $G(y) = P(Y < y) = P((X-1)^3 < y)$ $= P(X < 1 + y^{1/3}) = F(1 + y^{1/3})$ $= (1 + y^{1/3})^2/8 - (1 + y^{1/3})/4$ *or* $(y^{2/3} - 1)/8$	**(M1 A1)**
	OR: Use $x = 1 + y^{1/3}$ to find $f(x) = \frac{1}{4} y^{1/3}$ *and* $dx/dy = \frac{1}{3} y^{-2/3}$	**(M1 A1)**
	$g(y)\ [= G'(y)] = (1/12)\, y^{-1/3}$ *or* $1 / (12\, y^{1/3})$	**A1**
	for $1 \leqslant y \leqslant 27$ [$g(y) = 0$ otherwise]	**A1**
	Total:	**4**
(iii)	$(m^{2/3} - 1)/8 = \frac{1}{2}$	**M1**
	$m^{2/3} = 5$, $m = \sqrt{125}$ *or* $5\sqrt{5}$ *or* $11{\cdot}2$	**M1 A1**
	Total:	**3**

3 - (9231/23_Summer_2017_Q9) - *Further Work On Distributions*

(i)	$(a / \ln 2)\, [-\mathrm{e}^{-x \ln 2}]_0^{\infty} = a / \ln 2$ so $a = \ln 2$ *or* 0·693	**M1 A1**
	Total:	**2**
(ii)	$E(X) = 1 / \ln 2$ *or* 1·44	**B1**
	Total:	**1**

(iii)	$F(Q) = 1 - \mathrm{e}^{-Q \ln 2} = \frac{1}{4}$ *or* $\frac{3}{4}$	**M1**
	$Q_1 = (\ln 4/3) / (\ln 2)$ [= 0·415] (AEF)	**A1**
	$Q_3 = (\ln 4) / (\ln 2)$ [= 2] (AEF)	**A1**
	$Q_3 - Q_1$ [= (ln 3) / (ln 2)] = 1·58 [*or* 1·59]	**A1**
	Total:	**4**
(iv)	*EITHER:* $G(y) = P(Y < y) = P(2^X < y)$ $= P(X < (\ln y) / (\ln 2))$ $= F((\ln y) / (\ln 2))$ *or* $F(\log_2 y)$ (AEF)	**(M1 A1**
	$= 1 - \mathrm{e}^{-\ln y}$ *or* $1 - 1/y$	**A1)**
	OR: Use $x = (\ln y) / (\ln 2)$ to find *both*	**(M1**
	$f(x) = (\ln 2)\, \mathrm{e}^{-x \ln 2} = (\ln 2)\, \mathrm{e}^{-\ln y} = (1/y) \ln 2$	**A1**
	and $\mathrm{d}x/\mathrm{d}y = 1 / (y \ln 2)$	**A1)**
	$g(y)$ [= $G'(y)$] = $1/y^2$	**A1**
	for $y \geqslant 1$ [$g(y) = 0$ otherwise]	**A1**
	Total:	**5**

4 - (9231/21_Winter_2017_Q6) - *Further Work On Distributions*

(i)	$p = (1/6)^2$ *or* 1/36	**B1**	Find (or imply) probability p of pair of sixes in one throw
	$1/p = 36$	**B1**	Find mean value of X
(ii)	$P(X = 12) = p\,(1 - p)^{11} = 0{\cdot}0204$	**M1 A1**	Find prob. of needing exactly 12 throws
(iii)	$P(X > 12) = (1 - p)^{12} = 0{\cdot}713$	**M1 A1**	Find prob. of needing more than 12 throws
		2	

5 - (9231/21_Winter_2017_Q7) - *Further Work On Distributions*

(i)	$F(x) = \int f(x)\,dx = -e^{-0.2x} + c = 1 - e^{-0.2x}$ $(x \geq 0)$	M1	State, or integrate and use F(0) = 0 or $F(x) \to 1$ as $x \to \infty$
	and $F(x) = 0$ ($x < 0$ *or* otherwise)	A1	to find, F(x) (**A0** if case $x < 0$ omitted)
		2	
(ii)	$P(X > 2) = 1 - F(2) = e^{-0.4} = 0.670$	M1 A1	Find $P(X > 2)$: (**M0** for F(2))
		3	
(iii)	$1 - e^{-0.2m} = ½$, $e^{0.2m} = 2$	M1	Find median value m from F(m) *or* 1 – F(m) = ½
	$m = 5 \ln 2$ *or* 3·47	M1 A1	
		3	

6 - (9231/21_Summer_2018_Q6) - *Further Work On Distributions*

(i)	$P(X > 2) = 1 - F(2) = \exp(-0.8) = 0.449$	M1 A1	Find $P(X > 2)$. M0 for F(2)
		2	
(ii)	$F(Q) = 1 - \exp(-0.4x) = \frac{1}{4}$ *or* $\frac{3}{4}$	M1	Formulate equation for either quartile value Q
	$Q_1 = (\ln 4/3) / (0.4)$ [= 0·7192] (AEF)	A1	Find one [lower] quartile Q_1
	$Q_3 = (\ln 4) / (0.4)$ [= 3·466] (AEF)	A1	Find other [upper] quartile Q_3
	$Q_3 - Q_1$ [= (ln 3) / (0·4)] = 2·75	A1√	Find interquartile range (FT on Q_1, Q_3; allow $Q_1 - Q_3$)
		4	

7 - (9231/23_Summer_2018_Q7) - *Further Work On Distributions*

(i)	$(1 - p)/p^2 = 3.75$, $15p^2 + 4p - 4 = 0$ **AG**	M1 A1	Find given eqn. for p using $\text{Var}(X) = (1 - p)/p^2$
	$(5p - 2)(3p + 2) = 0$, $p = 2/5$ *or* 0·4	M1 A1	Solve quadratic for p (A0 if $p = -⅔$ not [implicitly] rejected)
		4	
(ii)	$P(X = 5) = (1 - p)^4 p = 0.6^4 \times 0.4 = 0.0518$ *or* 162/3125	B1	Find $P(X = 5)$
		1	
(iii)	*EITHER*: $P(3 \leqslant X \leqslant 7) = (1 - p)^2 - (1 - p)^7$	M1	Find $P(3 \leqslant X \leqslant 7)$
	$= 0.6^2 - 0.6^7 = 0.36 - 0.028 = 0.332$	A1	M0 for $P(X \leqslant 7) - P(X \leqslant 3)$ [= 0·188] or similar error
	OR: $P(3 \leqslant X \leqslant 7) = \sum_{r=3}^{7} (1 - p)^{r-1} p$	(M1)	
	$= (0.6^2 + 0.6^3 + 0.6^4 + 0.6^5 + 0.6^6) \times 0.4$ $= 0.830016 \times 0.4 = 0.332$	(A1)	
		2	

8 - (9231/21_Summer_2018_Q9) - *Further Work On Distributions*

(i)	$(1 - p)/p^2 = 4/9$, $4p^2 + 9p - 9 = 0$ **AG**	M1 A1	Find given eqn. for p using $\text{Var}(X) = (1 - p)/p^2$
	$(4p - 3)(p + 3) = 0$, $p = ¾$	M1 A1	Solve quadratic for p (A0 if $p = -3$ not [implicitly] rejected)
		4	
(ii)	$P(X = 3) = (1 - p)^2 p = (¼)^2 ¾ = 3/64$ *or* 0·0469	B1	Find $P(X = 3)$
		1	