A LEVEL Cambridge Topical Past Papers

FURTHER MECHANICS

2017 — 2023

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FURTHER MATHEMATICS P3 9231

TOPICAL PAST PAPER WORKSHEETS

2017 - 2023 | Questions + Mark scheme

AVAILABLE PAPERS

P1,2	P3	P4
395 Questions	179 Questions	164 Questions

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1 - (9231/21_Summer_2017_Q1) - *Momentum And Impulse*

A bullet of mass 0.08 kg is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed $300\,\text{m s}^{-1}$ and emerges horizontally after 0.02 s. There is a constant horizontal resisting force of magnitude 1000 N. Find the speed with which the bullet emerges from the barrier. [3]

2 - (9231/21_Summer_2017_Q3) - *Momentum And Impulse*

Two uniform small smooth spheres A and B have equal radii and masses $3m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Find, in terms of u and e, expressions for the velocities of A and B after the collision. [3]

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Sphere B continues to move until it strikes a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}$. When the spheres subsequently collide, A is brought to rest.

(ii) Find the value of e. [7]

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3 - (9231/23_Summer_2017_Q3) - *Momentum And Impulse*

Two uniform small smooth spheres A and B have equal radii and each has mass m. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$. Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A. The coefficient of restitution between B and the wall is $\frac{1}{3}$.

(i) Show that the speed of B after its collision with the wall is $\frac{5}{18}u$. [4]

(ii) Find the distance of B from the wall when it collides with A for the second time. [6]

4 - (9231/21_Winter_2017_Q3) - *Momentum And Impulse*

Three uniform small smooth spheres A, B and C have equal radii and masses m, km and m respectively, where k is a constant. The spheres are moving in the same direction along a straight line on a smooth horizontal surface, with B between A and C. The speeds of A, B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $\frac{1}{2}$. After sphere A has collided with sphere B, sphere B collides with sphere C.

(i) Find an inequality satisfied by k. [5]

(ii) Given that $k = 2$, show that after B has collided with C there are no further collisions between any of the three spheres. [5]

ANSWERS

1 - (9231/21_Summer_2017_Q1) - *Momentum And Impulse*

$0{\cdot}08 \times (300 - v) = 1000 \times 0{\cdot}02$ (AEF)	**M1 A1**
$v = 300 - 250 = 50$ [m s^{-1}]	**A1**
Total:	**3**

2 - (9231/21_Summer_2017_Q3) - *Momentum And Impulse*

(i)	$3mv_A + mv_B = 3mu$, $v_B - v_A = eu$ (AEF)	**M1**
	$v_A = \frac{1}{4}(3 - e)u$, $v_B = \frac{3}{4}(1 + e)u$	**A1, A1**
	Total:	**3**
(ii)	$v_B' = -\frac{3}{4} v_B$ [$= -(9/16)(1 + e)u$] (AEF)	**B1**
	[$3mV_A$ +] $mV_B = 3mv_A + mv_B'$ [$V_B = 3(9 - 7e)u/16$]	**M1**
	V_B [$- V_A$] $= -e(v_B' - v_A)$ [$V_B = e(21 + 5e)u/16$]	**M1**
	EITHER: [$4V_A$ =] $(3 - e)v_A + (1 + e)v_B' = 0$ $\frac{1}{4}(3 - e)^2 - (9/16)(1 + e)^2 = 0$ (AEF)	**(M1 A1)**
	OR: $3(9 - 7e) = e(21 + 5e)$	**(M1 A1)**
	$5e^2 + 42e - 27 = 0$, $e = 3/5$ *or* $0{\cdot}6$	**M1 A1**
	Total:	**7**

3 - (9231/23_Summer_2017_Q3) - *Momentum And Impulse*

(i)	$mv_A + mv_B = mu$ (AEF)	***M1**
	$v_B - v_A = \frac{2}{3}u$	***M1**
	$v_B = 5u/6$	**A1**
	$w_B = \frac{1}{3} v_B = 5u/18$ AG	**B1**
	Total:	**4**

(ii)	$v_A = u / 6$	**DA1**
	EITHER: $(d - x) / v_A = d / v_B + x / w_B$ (AEF)	**(M1 A1**
	$6(d - x) = 1{\cdot}2\, d + 3{\cdot}6\, x$	**M1 A1)**
	OR: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	**(M1**
	$t_2 = (0{\cdot}8\, d) / (v_A + w_B) = 9d/5u$	**M1 A1**
	$y_A = v_A\, t_2 = 0{\cdot}3\, d$ *or* $y_B = w_B\, t_2 = 0{\cdot}5\, d$	**A1)**
	OR2: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	**(M1**
	$(0{\cdot}8\, d - x) / v_A = x / w_B$ *or* $0{\cdot}8\, d/(v_A + w_B) = x/w_B$	**M1 A1**
	$4{\cdot}8\, d - 6\, x = 3{\cdot}6\, x$ *or* $1{\cdot}8\, d = 3{\cdot}6\, x$	**A1)**
	$x = ½\, d$	**A1**
	Total:	**6**

4 - (9231/21_Winter_2017_Q3) - *Momentum And Impulse*

(i)	$mv_A + kmv_B = 2mu + kmu$ (AEF) $[v_A + kv_B = 2u + ku]$	M1	Use conservation of momentum for *A* & *B* (allow omission of *m* in all momentum eqns)
	$v_B - v_A = ½\,(2u - u)\ [= ½\, u]$	M1	Use Newton's restitution law with consistent LHS signs
	$v_B = u\,(2k + 5) / 2(k + 1)$ *or* $u\,(k + 5/2) / (k + 1)$ (AEF)	A1	Combine to find v_B
	$[v_A = u\,(k + 4) / 2(k + 1)]\ v_B > 4u/3$ if $k < 7/2$	M1 A1	Find inequality for *k* from speeds of *B* and *C* after 1st collision
		5	
(ii)	$kmw_B + mv_C = kmv_B + m(4u/3)$ (AEF) $[2w_B + v_C = 2(3u/2) + 4u/3$ $= 13u/3$ when $k = 2]$	M1	Use conservation of momentum for *B* & *C*
	$v_C - w_B = ½\,(v_B - 4u/3)\ [= u/12]$ $(k + 1)\, w_B = (k - ½)\, v_B + 2u$	M1	Use Newton's restitution law with consistent LHS signs Combine to find w_B
	$3\, w_B = (3/2)\, v_B + 2u$ with $v_B = 3u/2$, so $w_B = 17u/12$	*A1	when $k = 2$
	$v_A = u,\ v_A < w_B$	DB1	Verify no further collisions between *A* and *B*
	EITHER: $(k + 1)\, v_C = (3k/2)\, v_B + (2 - k)\,(2u/3)$ $3\, v_C = 3\, v_B$ with $v_B = 3u/2$ so $v_C = 3u/2 > w_B$	(DB1)	*EITHER:* Find v_C and verify no further collisions between *B* and *C*
	OR: *B* and *C* cannot meet again since they move apart after colliding (AEF)	(DB1)	*OR:* State explicitly that no further collisions between *B* and *C*
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