A LEVEL Cambridge Topical Past Papers

FURTHER MATHEMATICS

P1,P2

2017 — 2023

FURTHER MATHEMATICS P1,P2 9231

TOPICAL PAST PAPER WORKSHEETS

2017 - 2023 | Questions + Mark scheme

AVAILABLE PAPERS

P1,2	P3	P4
395 Questions	179 Questions	164 Questions

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TOPICS	P1, P2
Roots Of Polynomial Equations	26
Rational Functions And Graphs	27
Summation Of Series	34
Matrices	61
Polar Coordinates	26
Vectors	27
Proof By Induction	25
Hyperbolic Functions	17
Differentiation	31
Integration	51
Complex Numbers	31
Differential Equations	39

1 - (9231/11_Summer_2017_Q7) - *Roots Of Polynomial Equations*

By finding a cubic equation whose roots are α, β and γ, solve the set of simultaneous equations

$$\alpha + \beta + \gamma = -1,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 29,$$
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -1.$$

[8]

2 - (9231/13_Summer_2017_Q1) - *Roots Of Polynomial Equations*

The roots of the cubic equation $x^3 + 2x^2 - 3 = 0$ are α, β and γ.

(i) By using the substitution $y = \frac{1}{x^2}$, find the cubic equation with roots $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$. [3]

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(ii) Hence find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$. [1]

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(iii) Find also the value of $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$. [1]

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3 - (9231/11_Winter_2017_Q4) - *Roots Of Polynomial Equations*

The cubic equation $2x^3 - 3x^2 + 4x - 10 = 0$ has roots α, β and γ.

(i) Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4]

(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. [4]

4 - (9231/11_Summer_2018_Q4) - *Roots Of Polynomial Equations*

It is given that the equation

$$x^3 - 21x^2 + kx - 216 = 0,$$

where k is a constant, has real roots a, ar and ar^{-1}.

(i) Find the numerical values of the roots. [6]

(ii) Deduce the value of k. [2]

5 - (9231/13_Summer_2018_Q6) - *Roots Of Polynomial Equations*

The equation

$$9x^3 - 9x^2 + x - 2 = 0$$

has roots α, β, γ.

(i) Use the substitution $y = 3x - 1$ to show that $3\alpha - 1$, $3\beta - 1$, $3\gamma - 1$ are the roots of the equation

$$y^3 - 2y - 7 = 0. \qquad [2]$$

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The sum $(3\alpha - 1)^n + (3\beta - 1)^n + (3\gamma - 1)^n$ is denoted by S_n.

(ii) Find the value of S_3. [2]

(iii) Find the value of S_{-2}. [4]

6 - (9231/11_Winter_2018_Q2) - *Roots Of Polynomial Equations*

The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are α, 2α, 4α, where p, q, r and α are non-zero real constants.

(i) Show that

$$2p\alpha + q = 0.$$ [4]

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(ii) Show that

$$p^3 r - q^3 = 0.$$ [2]

ANSWERS

1 - (9231/11_Summer_2017_Q7) - *Roots Of Polynomial Equations*

$2\sum\alpha\beta = 1-29 \Rightarrow \sum\alpha\beta = -14$	**M1A1**
$\frac{\sum\alpha\beta}{\alpha\beta\gamma} = \frac{-14}{\alpha\beta\gamma} = -1 \Rightarrow \alpha\beta\gamma = 14$	**M1A1 FT**
$\Rightarrow x^3 + x^2 - 14x - 14 = 0$	**A1**
$\Rightarrow (x+1)(x^2-14)$	**M1A1**
⇒Solution is –1, in $\pm\sqrt{14}$ any order. Accept ±3.74 (awrt) SR **B1** for correct roots without working	**A1**
Total:	**8**

2 - (9231/13_Summer_2017_Q1) - *Roots Of Polynomial Equations*

(i)	$y = \frac{1}{x^2} \Rightarrow x = \frac{1}{\sqrt{y}}$	**M1**
	$\frac{1}{y\sqrt{y}} + \frac{2}{y} - 3 = 0 \Rightarrow \frac{1}{y\sqrt{y}} = 3 - \frac{2}{y}$	**M1**
	$\Rightarrow 9y^3 - 12y^2 + 4y - 1 = 0$ SR **B1** for finding cubic by manipulating roots	**A1**
	Total:	**3**
(ii)	$\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{12}{9}$ or $\frac{4}{3}$	**B1FT**
	Total:	**1**
(iii)	$\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} = \frac{4}{9}$	**B1FT**
	Total:	**1**

3 - (9231/11_Winter_2017_Q4) - *Roots Of Polynomial Equations*

(i)	$\alpha+\beta+\gamma=\frac{3}{2}$ $\quad \alpha\beta+\beta\gamma+\gamma\alpha=2$ $\quad \alpha\beta\gamma=5+\beta+\gamma=$ $\frac{3}{2}\alpha\beta+\beta\gamma+\gamma\alpha=2\alpha\beta\gamma=5$	B1	(Can be awarded in (ii) if not seen here) SOI
	$(\alpha+1)(\beta+1)(\gamma+1)=\alpha\beta\gamma+(\alpha\beta+\beta\gamma+\gamma\alpha)+$ $(\alpha+\beta+\gamma)+1$	M1A1	Multiply out and group for M1
	$=5+2+1\frac{1}{2}+1=9\frac{1}{2}$	A1FT	Alt method: Let $x=y-1$ M1 Sub and expand $2y^3-9y^2\ 16y-19=0$ M1, A1 Product of roots = 19/2 A1
		4	
(ii)	$(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)=\left(1\frac{1}{2}-\alpha\right)\left(1\frac{1}{2}-\beta\right)\left(1\frac{1}{2}-\gamma\right)$	M1	Alt methods: $=(\sum\alpha)(\sum\alpha\beta)-\alpha\beta\gamma$ or $\sum\alpha^2\sum\alpha+2\alpha\beta\gamma-\sum\alpha^3$
	$=\frac{27}{8}-\frac{9}{4}(\alpha+\beta+\gamma)+\frac{3}{2}(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$	A1	
	$=\frac{27}{8}-\frac{9}{4}\times\frac{3}{2}+\frac{3}{2}\times2-5=-2$	M1A1	
		4	

4 - (9231/11_Summer_2018_Q4) - *Roots Of Polynomial Equations*

(i)	$\alpha\beta\gamma=a^3=216\Rightarrow a=6$	M1 A1	Uses product of roots
	$a+ar+ar^{-1}=21$ $6(1+r+r^{-1})=21$	M1	Uses sum of roots
	$2r^2-5r+2=0\Rightarrow r=2$ or $r=0.5$	M1 A1	Substitutes for a and solves quadratic
	Roots are 6, 12, 3	A1	
		6	
(ii)	$k=\alpha\beta+\alpha\gamma+\beta\gamma=6(12)+6(3)+12(3)=126$	M1 A1	Or finds coefficient of x in $(x-3)(x-6)(x-12)$. Or substitutes root into equation
		2	

5 - (9231/13_Summer_2018_Q6) - *Roots Of Polynomial Equations*

(i)	Substitutes $x = \frac{y+1}{3}$	M1	Accept substitution of $y = 3x - 1$ into given equation and derivation of equation in x.
	Obtains the given result	A1	AG.
(ii)	$S_3 = 2S_1 + 7 \times 3$	M1	Uses $y^3 = 2y + 7$. Or uses formula for $\Sigma\,(3\alpha - 1)^3$
	$= 21$	A1	
(iii)	$S_{-1} = \frac{(3\alpha-1)(3\beta-1)+(3\alpha-1)(3\gamma-1)+(3\beta-1)(3\gamma-1)}{(3\alpha-1)(3\beta-1)(3\gamma-1)} = \frac{-2}{7}$.	M1 A1	Award M1A1 if $S_{-1} = -\frac{2}{7}$ written down directly.
	$7S_{-2} = S_1 - 2S_{-1}$	M1	Uses $7y^{-2} = y - 2y^{-1}$.
	$s_{-2} = \frac{4}{49}$	A1	
	Alt method: $S_{-2} = \Sigma\frac{1}{(3\alpha-1)^2} = \frac{\Sigma(3\alpha-1)^2(3\beta-1)^2}{(3\alpha-1)^2(3\beta-1)^2(3\gamma-1)^2} =$	M1 A1	**Alt method**: Finds cubic with roots $\frac{1}{3\alpha-1}$, etc. M1 $7z^3 + 2z^2 - 1 = 0$ A1 Uses $S_2 = (S_1)^2 - 2\times\Sigma\alpha\beta$ M1 $= \frac{4}{49}$ A1
	$\frac{(\Sigma(3\alpha-1)(3\beta-1))^2 - 2(3\alpha-1)(3\beta-1)(3\gamma-1)(\Sigma(3\alpha-1))}{(3\alpha-1)^2(3\beta-1)^2(3\gamma-1)^2}$	M1	
	$= \frac{(-2)2 - 2(7)(0)}{7^2} = \frac{4}{49}$	A1	
		8	

6 - (9231/11_Winter_2018_Q2) - *Roots Of Polynomial Equations*

(i)	$\alpha + 2\alpha + 4\alpha = -p$	B1	Sum of roots.
	$2\alpha^2 + 4\alpha^2 + 8\alpha^2 = q$	B1	Sum of products in pairs.
	$\frac{14\alpha^2}{7\alpha} = -\frac{q}{p}$	M1	Combines equations.
	$\Rightarrow 2p\alpha + q = 0$	A1	Verifies result (AG).
		4	
(ii)	$8\alpha^3 = -r$	B1	Product of roots.
	$\Rightarrow r = \frac{q^3}{p^3} \Rightarrow p^3 r - q^3 = 0$	B1	Verifies result (AG).
		2	

7 - (9231/12_Winter_2018_Q1) - *Roots Of Polynomial Equations*

(i)	$\alpha + \beta + \gamma = 5,\ \alpha\beta + \alpha\gamma + \beta\gamma = 13$	B1	Sum of roots and $\alpha\beta + \alpha\gamma + \beta\gamma$. SOI
	$\alpha^2 + \beta^2 + \gamma^2 = 5^2 - 2(13)$	M1	Uses $\Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)$
	$= -1$	A1	www
		3	
(ii)	$\alpha^3 + \beta^3 + \gamma^3 = 5(\alpha^2 + \beta^2 + \gamma^2) - 13(\alpha + \beta + \gamma) + 12$	M1	Uses $\alpha^3 = 5\alpha^2 - 13\alpha + 4$.
	$= 5(-1) - 13(5) + 12 = -58$	A1	
	Alt method: Use formula e.g. $\Sigma\alpha^3 = (\Sigma\alpha)(\Sigma\alpha^2 - \Sigma\alpha\beta) + 3\alpha\beta\gamma$ Or $(\Sigma\alpha)^3 - 3(\Sigma\alpha)(\Sigma\alpha\beta) + 3\alpha\beta\gamma$		
		2	