

A LEVEL Cambridge Topical Past Papers

# PURE MATHEMATICS 1

2017 — 2023

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1 - (9709/11\_Summer\_2017\_Q5) - Trigonometry, Coordinates Geometry

The equation of a curve is  $y = 2 \cos x$ .

- (i) Sketch the graph of  $y = 2 \cos x$  for  $-\pi \leq x \leq \pi$ , stating the coordinates of the point of intersection with the  $y$ -axis. [2]

Points  $P$  and  $Q$  lie on the curve and have  $x$ -coordinates of  $\frac{1}{3}\pi$  and  $\pi$  respectively.

- (ii) Find the length of  $PQ$  correct to 1 decimal place. [2]

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The line through  $P$  and  $Q$  meets the  $x$ -axis at  $H(h, 0)$  and the  $y$ -axis at  $K(0, k)$ .

(iii) Show that  $h = \frac{5}{9}\pi$  and find the value of  $k$ .

[3]

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2 - (9709/12\_Summer\_2017\_Q2) - *Coordinates Geometry*

The point  $A$  has coordinates  $(-2, 6)$ . The equation of the perpendicular bisector of the line  $AB$  is  $2y = 3x + 5$ .

(i) Find the equation of  $AB$ . [3]

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(ii) Find the coordinates of  $B$ . [3]

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3 - (9709/13\_Summer\_2017\_Q8) - *Coordinates Geometry*

$A(-1, 1)$  and  $P(a, b)$  are two points, where  $a$  and  $b$  are constants. The gradient of  $AP$  is 2.

(i) Find an expression for  $b$  in terms of  $a$ . [2]

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(ii)  $B(10, -1)$  is a third point such that  $AP = AB$ . Calculate the coordinates of the possible positions of  $P$ . [6]

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4 - (9709/11\_Winter\_2017\_Q6) - *Coordinates Geometry*

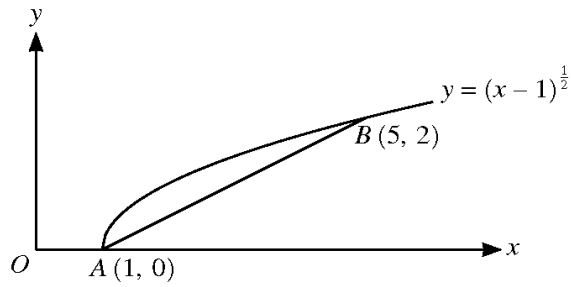
The points  $A(1, 1)$  and  $B(5, 9)$  lie on the curve  $6y = 5x^2 - 18x + 19$ .

(i) Show that the equation of the perpendicular bisector of  $AB$  is  $2y = 13 - x$ . [4]

The perpendicular bisector of  $AB$  meets the curve at  $C$  and  $D$ .

(ii) Find, by calculation, the distance  $CD$ , giving your answer in the form  $\sqrt{\left(\frac{p}{q}\right)}$ , where  $p$  and  $q$  are integers. [5]

5 - (9709/13\_Winter\_2017\_Q11) - Coordinates Geometry, Differentiation



The diagram shows the curve  $y = (x - 1)^{\frac{1}{2}}$  and points  $A(1, 0)$  and  $B(5, 2)$  lying on the curve.

(i) Find the equation of the line  $AB$ , giving your answer in the form  $y = mx + c$ . [2]

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(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to  $AB$ . [5]

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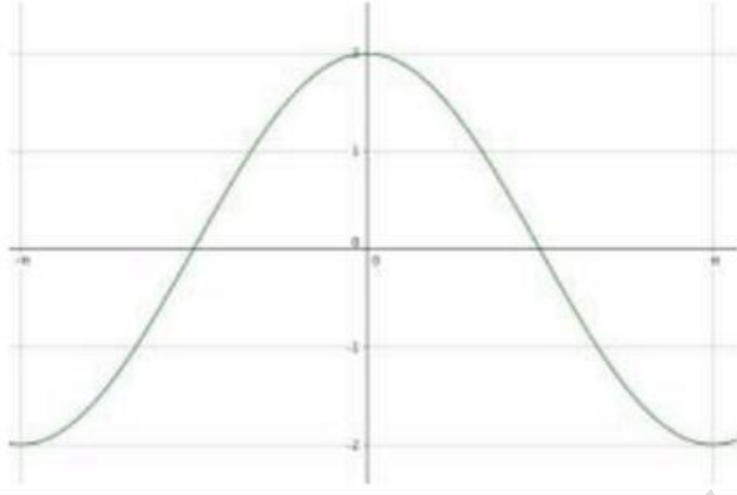
- (iii) Find the perpendicular distance between the line  $AB$  and the tangent parallel to  $AB$ . Give your answer correct to 2 decimal places. [3]

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# ANSWERS

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1 - (9709/11\_Summer\_2017\_Q5) - Trigonometry, Coordinates Geometry

(i)		<b>B1</b>
		<b>DB1</b>
<b>Total:</b>		<b>2</b>
(ii)	$P\left(\frac{\pi}{3}, 1\right) Q(\pi, -2)$	
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	<b>M1 A1</b>
<b>Total:</b>		<b>2</b>
(iii)	Eqn of $PQ$ $y-1 = -\frac{9}{2\pi}\left(x-\frac{\pi}{3}\right)$	<b>M1</b>
	If $y=0 \rightarrow h = \frac{5\pi}{9}$	<b>A1</b>
	If $x=0 \rightarrow k = \frac{5}{2}$ ,	<b>A1</b>
	<b>Total:</b>	<b>3</b>

## 2 - (9709/12\_Summer\_2017\_Q2) - Coordinates Geometry

(i)	Gradient = 1.5 Gradient of perpendicular = $-\frac{2}{3}$	B1
	Equation of AB is $y - 6 = -\frac{2}{3}(x + 2)$ Or $3y + 2x = 14$ oe	M1 A1
	<b>Total:</b>	<b>3</b>
(ii)	Simultaneous equations $\rightarrow$ Midpoint (1, 4)	M1
	Use of midpoint or vectors $\rightarrow B(4, 2)$	M1A1
	<b>Total:</b>	<b>3</b>

## 3 - (9709/13\_Summer\_2017\_Q8) - Coordinates Geometry

(i)	$(b-1)/(a+1) = 2$	M1
	$b = 2a + 3$ CAO	A1
	<b>Total:</b>	<b>2</b>
(ii)	$AB^2 = 11^2 + 2^2 = 125$ oe	B1
	$(a+1)^2 + (b-1)^2 = 125$	B1 FT
	$(a+1)^2 + (2a+2)^2 = 125$	M1
	$(5)(a^2 + 2a - 24) = 0 \rightarrow \text{eg}(a-4)(a+6) = 0$	M1
	$a = 4$ or $-6$	A1
	$b = 11$ or $-9$	A1
	<b>Total:</b>	<b>6</b>

## 4 - (9709/11\_Winter\_2017\_Q6) - Coordinates Geometry

(i)	Mid-point of $AB = (3, 5)$	B1	Answers may be derived from simultaneous equations
	Gradient of $AB = 2$	B1	
	Eqn of perp. bisector is $y - 5 = -\frac{1}{2}(x - 3) \Rightarrow 2y = 13 - x$	M1A1	AG For M1 FT from mid-point and gradient of $AB$
		4	
(ii)	$-3x + 39 = 5x^2 - 18x + 19 \rightarrow (5)(x^2 - 3x - 4) = 0$	M1	Equate equations and form 3-term quadratic
	$x = 4$ or $-1$	A1	
	$y = 4\frac{1}{2}$ or $7$	A1	
	$CD^2 = 5^2 + 2\frac{1}{2}^2 \rightarrow CD = \sqrt{\frac{125}{4}}$	M1A1	Or equivalent integer fractions ISW
	5		

## 5 - (9709/13\_Winter\_2017\_Q11) - Coordinates Geometry, Differentiation

(i)	Gradient of $AB = \frac{1}{2}$	B1	
	Equation of $AB$ is $y = \frac{1}{2}x - \frac{1}{2}$	B1	
		2	
(ii)	$\frac{dy}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}}$	B1	
	$\frac{1}{2}(x-1)^{-\frac{1}{2}} = \frac{1}{2}$ . Equate their $\frac{dy}{dx}$ to their $\frac{1}{2}$	*M1	
	$x = 2, y = 1$	A1	
	$y - 1 = \frac{1}{2}(x - 2)$ (thro' their(2,1) & their $\frac{1}{2}$ ) $\rightarrow y = \frac{1}{2}x$	DM1 A1	
	5		

(iii)	<i>EITHER:</i> $\sin \theta = \frac{d}{1} \rightarrow d = \sin \theta$	(M1)	Where $\theta$ is angle between $AB$ and the $x$ -axis
	gradient of $AB = \frac{1}{2} \Rightarrow \tan \theta = \frac{1}{2} \Rightarrow \theta = 26.5(7)^\circ$	B1	
	$d = \sin 26.5(7)^\circ = 0.45$ (or $\frac{1}{\sqrt{5}}$ )	A1)	
	<i>OR1:</i> Perpendicular through $O$ has equation $y = -2x$	(M1)	
	Intersection with $AB$ : $-2x = \frac{1}{2}x - \frac{1}{2} \rightarrow \left(\frac{1}{5}, -\frac{2}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$ )	A1)	
	<i>OR2:</i> Perpendicular through $(2, 1)$ has equation $y = -2x + 5$	(M1)	
	Intersection with $AB$ : $-2x + 5 = \frac{1}{2}x - \frac{1}{2} \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{11}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$ )	A1)	