IGCSE (9-1) Edexcel Past Papers

FURTHER PURE MATHEMATICS

Paper 1, 1R

2019 - 2023

1 - (4PM1/1_Summer_2019_Q8) - *Logarithmic Functions And Indices*

(a) Solve $5p^2 - 9p + 4 = 0$

(2)

(b) Hence solve $5^{2x+1} - 9(5^x) + 4 = 0$

Give your answers to 3 significant figures where appropriate.

(4)

The curve with equation $y = 5^{2x+1} + 5^x$ intersects the curve with equation $y = 2(5^{x+1}) - 4$ at two points.

(c) Find the coordinates of each of these two points.
Give your answers to 3 significant figures where appropriate.

(4)

2 - (4PM1/1_Summer_2019_Q9) - *Logarithmic Functions And Indices*

(a) Solve the equation $2\log_p 9 + 3\log_3 p = 8$

(6)

Given that $\log_2 3 = \log_4 3^k$

(b) find the value of k

(2)

(c) Show that

$$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3 = \log_4\left(\frac{x^{6x-5}}{3^{6x-20}}\right)$$

(4)

3 - (4PM1/1R_Summer_2019_Q3) - *Logarithmic Functions And Indices*

(a) Write down the value of $\log_3 9$

(1)

(b) Solve the equation $\log_3 9t = \log_9 \left(\frac{12}{t}\right)^2 + 2$ where $t > 0$

Give your answer in the form $a\sqrt{b}$ where a and b are prime numbers.

(6)

4 - (4PM1/1R_Summer_2019_Q4) - *Logarithmic Functions And Indices*

$$f(x) = e^{3x}\sqrt{1+2x}$$

(a) Show that $$f'(x) = \frac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$$

(4)

(b) Find an equation of the normal to the curve with equation $y = f(x)$ at the point on the curve where $x = 0$

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

5 - (4PM1/1R_Summer_2020_Q9) - *Logarithmic Functions And Indices*

Showing your working clearly, use algebra to solve the equations

$$\frac{16^x}{8^y} = \frac{1}{4}$$

$$4^x 2^y = 16$$

(7)

ANSWERS

1 - (4PM1/1_Summer_2019_Q8) - *Logarithmic Functions And Indices*

(a)	$(5p-4)(p-1)=0$ $p=\frac{4}{5},\ p=1$	M1A1 [2]
(b)	$5^{2x+1}-9(5^x)+4=0 \Rightarrow 5\times 5^{2x}-9(5^x)+4=0$ $5^x=1\ \ x=0$ $5^x=\frac{4}{5},\ x\ln 5=\ln\left(\frac{4}{5}\right)\ \ x=-0.1386...=-0.139$	M1 A1 M1A1 [4]
(c)	$5^{2x+1}+5^x=2(5^{x+1})-4$ $5^{2x+1}-9(5^x)+4=0$ $x=0\ \ y=5+1=6\ (\text{or } y=2\times 5-4=6)\quad (0,6)$ $x=-0.1386...\ \ 5^x=\frac{4}{5}\ \ y=5\times\left(\frac{4}{5}\right)^2+\frac{4}{5}=4\ \left(\text{or } y=10\times\frac{4}{5}-4=4\right)$ $(-0.139,4)$	M1 A1 M1 A1 [4]

2 - (4PM1/1_Summer_2019_Q9) - *Logarithmic Functions And Indices*

(a)	$2\log_p 9 + 3\log_3 p = 8$	
	$2\dfrac{\log_3 9}{\log_3 p} + 3\log_3 p = 8$	M1
	$2\log_3 9 + 3(\log_3 p)^2 = 8\log_3 p$	M1
	$3(\log_3 p)^2 - 8\log_3 p + 4 = 0$	A1
	$(3\log_3 p - 2)(\log_3 p - 2) = 0$	M1
	$\log_3 p = \dfrac{2}{3} \quad p = 3^{\frac{2}{3}} = \sqrt[3]{9} \quad (= 2.08)$	A1
	$\log_3 p = 2 \quad p = 3^2 = 9$	A1 [6]
(b)	$\log_2 3 = \dfrac{\log_4 3}{\log_4 2} = \dfrac{\log_4 3}{\frac{1}{2}} = 2\log_4 3 = \log_4 3^2 \Rightarrow k = 2$	M1A1 [2]
(c)	$6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3$	
	$= 6x\log_4 x - 5\log_4 x - 3x\log_4 3^2 + 10\log_4 3^2$	M1
	$= \log_4 x^{6x} - \log_4 x^5 - \log_4 3^{6x} + \log_4 3^{20}$	M1
	$= \log_4 \dfrac{x^{6x} \times 3^{20}}{x^5 \times 3^{6x}}$	M1
	$= \log_4 \dfrac{x^{6x-5}}{3^{6x-20}}$ *	A1 [4]

3 - (4PM1/1R_Summer_2019_Q3) - *Logarithmic Functions And Indices*

(a)	$\log_3 9 = 2$	B1 [1]
(b)	$\log_3 9t = \log_9 \left(\dfrac{12}{t}\right)^2 + 2 \Rightarrow \log_3 9 + \log_3 t = 2(\log_9 12 - \log_9 t) + 2$	M1M1
	$\log_3 9 + \log_3 t = 2\left(\dfrac{\log_3 12}{\log_3 9} - \dfrac{\log_3 t}{\log_3 9}\right) + 2$	M1
	$\Rightarrow \log_3 9 + \log_3 t = \log_3 12 - \log_3 t + 2$	
	$\Rightarrow 2\log_3 t = \log_3 12 \Rightarrow \log_3 t^2 = \log_3 12$	A1
	$\Rightarrow t^2 = 12 \Rightarrow t = 2\sqrt{3}$	M1A1 [6]

4 - (4PM1/1R_Summer_2019_Q4) - *Logarithmic Functions And Indices*

(a)	$f'(x)=3e^{3x}(1+2x)^{\frac{1}{2}}+e^{3x}\times\frac{1}{2}\times2(1+2x)^{-\frac{1}{2}}$ $\Rightarrow f'(x)=\frac{3e^{3x}(1+2x)+e^{3x}}{\sqrt{1+2x}}\Rightarrow f'(x)=\frac{2e^{3x}(2+3x)}{\sqrt{1+2x}}$ *	M1A1 M1A1 [4]
(b)	When $x=0$ $f'(0)=\frac{2e^{0}(2+0)}{\sqrt{1+0}}=4$ Gradient of Normal $=-\frac{1}{4}$ $f(0)=e^{0}\sqrt{1+2\times0}=1$ Equation of Normal to curve $y=f(x)$ when $x=0$ $y-1=-\frac{1}{4}(x-0)$ $\Rightarrow x+4y-4=0$	B1B1 B1 M1A1 A1 [6]

5 - (4PM1/1R_Summer_2020_Q9) - *Logarithmic Functions And Indices*

	$\frac{2^{4x}}{2^{3y}} = \frac{1}{2^2}$	M1
	$2^{4x-2y} = 2^{-2}$ (→ $4x - 3y = -2$)	dM1
	$2^{2x}2^y = 2^4$	M1
	$2^{2x+y} = 2^4$ → ($2x + y = 4$)	dM1
	A fully correct method using for solving simultaneously leading to either $10x = 10$ **or** $5y = 10$ $4x - 3y = -2 \Rightarrow 10x = 10$ or $4x - 3y = -2 \Rightarrow 5y = 10$ $6x + 3y = 12$ $4x + 2y = 8$	ddddM1
	$y = 2$ $x = 1$	A1 A1 **[7]**
	Alternative Method $4^x = \frac{16}{2^y}$	M1
	$\frac{4^{2x}}{8^y} = \frac{1}{4}$	M1
	$\left(\frac{16}{2^y}\right)^2 \times \frac{1}{8^y} = \frac{1}{4}$	ddM1
	$8^y \times 2^{2y} = 4 \times 16^2$	dddM1
	$2^{3y} \times 2^{2y} = 2^2 \times 2^8$	ddddM1
	($2^{5y} = 2^{10}$) $y = 2$	A1
	($4^x \times 4 = 16$) $x = 1$	A1