

A-Level Edexcel

FURTHER PURE MATHEMATICS

UNIT F2(IAL)

2020 – 2023

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1 - (WFM01/F2(IAL)_Summer_2020_Q4) - Complex Numbers

(a) Express the complex number $18\sqrt{3} - 18i$ in the form

$$r(\cos \theta + i \sin \theta) \quad -\pi < \theta \leq \pi \quad (3)$$

(b) Solve the equation

$$z^4 = 18\sqrt{3} - 18i$$

giving your answers in the form $re^{i\theta}$ where $-\pi < \theta \leq \pi$ (5)

2 - (WFM01/F2(IAL)_Summer_2020_Q5) - *Complex Numbers*

The transformation T from the z -plane to the w -plane is given by

$$w = \frac{z - 3i}{z + 2i} \quad z \neq -2i$$

The circle with equation $|z| = 1$ in the z -plane is mapped by T onto the circle C in the w -plane.

Determine

- (i) the centre of C ,
- (ii) the radius of C .

(7)

3 - (WFM01/F2(IAL)_Summer_2021_Q2) - *Complex Numbers*

The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by

$$w = \frac{z + 2}{z - i} \quad z \neq i$$

The transformation T maps the circle $|z| = 2$ in the z -plane onto a circle C in the w -plane.

- Find (i) the centre of C ,
(ii) the radius of C .

(8)

4 - (WFM01/F2(IAL)_Summer_2021_Q7) - *Complex Numbers*

(a) Use de Moivre's theorem to show that

$$\tan 4\theta \equiv \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} \quad (6)$$

(b) Use the identity given in part (a) to find the 2 positive roots of

$$x^4 + 2x^3 - 6x^2 - 2x + 1 = 0$$

giving your answers to 3 significant figures.

(3)

5 - (WFM01/F2(IAL)_Winter_2021_Q1) - Complex Numbers

The transformation T from the z -plane, where $z = x + iy$, to the w -plane, where $w = u + iv$, is given by

$$w = \frac{z + pi}{iz + 3} \quad z \neq 3i \quad p \in \mathbb{Z}$$

The point representing $i(1 + \sqrt{3})$ is invariant under T .

Determine the value of p .

(3)

6 - (WFM01/F2(IAL)_Winter_2021_Q8) - Complex Numbers

Given that $z = e^{i\theta}$

(a) show that $z^n + \frac{1}{z^n} = 2 \cos n\theta$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta = 0 \quad 0 \leq \theta \leq \pi$$

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} (32 \cos^6 \theta - 4 \cos^2 \theta) d\theta$$

Solutions relying entirely on calculator technology are not acceptable.

(5)

ANSWERS

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1 - (WFM01/F2(IAL)_Summer_2020_Q4) - Complex Numbers

(a)	$ 18\sqrt{3} - 18i = 18\sqrt{(3+1)} = 36$ $\tan \theta = \frac{-18}{18\sqrt{3}} \quad \theta = -\frac{\pi}{6}, \quad 18\sqrt{3} - 18i = 36 \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right)$	B1 M1,A1cao (3)
(b)	$z^4 = 36 \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6} \right) = 36 \left(\cos\left(2k\pi - \frac{\pi}{6}\right) + i \sin\left(2k\pi - \frac{\pi}{6}\right) \right)$ $z = \sqrt[4]{36} \left(\cos\left(\frac{12k\pi - \pi}{24}\right) + i \sin\left(\frac{12k\pi - \pi}{24}\right) \right)$ $k = 0 \quad z_0 = \sqrt[4]{36} \left(\cos\left(\frac{-\pi}{24}\right) + i \sin\left(\frac{-\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\left(\frac{-\pi}{24}\right)}$ $k = 1 \quad z_1 = \sqrt[4]{36} \left(\cos\left(\frac{11\pi}{24}\right) + i \sin\left(\frac{11\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\frac{11\pi}{24}}$ $k = 2 \quad z_2 = \sqrt[4]{36} \left(\cos\left(\frac{23\pi}{24}\right) + i \sin\left(\frac{23\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\frac{23\pi}{24}}$ $k = -1 \quad z_3 = \sqrt[4]{36} \left(\cos\left(-\frac{13\pi}{24}\right) + i \sin\left(-\frac{13\pi}{24}\right) \right) = \sqrt[4]{36} e^{i\left(-\frac{13\pi}{24}\right)}$	M1 M1 B1 A1ft A1ft (5) [8]

2 - (WFM01/F2(IAL)_Summer_2020_Q5) - Complex Numbers

	$w = \frac{z-3i}{z+2i}$ $w(z+2i) = z-3i \quad z = \frac{i(2w+3)}{1-w}$ $ z =1 \quad \left \frac{i(2w+3)}{1-w} \right = 1$ $ i(2w+3) = 1-w $ $w = u+iv \quad (2u+3)^2 + 4v^2 = (1-u)^2 + v^2$ $4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$ $3u^2 + 3v^2 + 14u + 8 = 0$ $u^2 + v^2 + \frac{14}{3}u + \frac{8}{3} = 0$ $\left(u + \frac{7}{3}\right)^2 + v^2 = -\frac{8}{3} + \frac{49}{9} = \frac{25}{9}$	M1 dM1 ddM1 dddM1 A1
(i)	Centre $\left(-\frac{7}{3}, 0\right)$	A1
(ii)	Radius $\frac{5}{3}$	A1 (7) [7]

3 - (WFM01/F2(IAL)_Summer_2021_Q2) - Complex Numbers

	$w = \frac{z+2}{z-i} \quad z \neq i$ $z = \frac{2+iw}{w-1}$ $ z = 2 \Rightarrow \left \frac{2+iw}{w-1} \right = 2 \Rightarrow 2+iw = 2 w-1 $ $ 2+iu-v = 2 u+iv-1 $ $(2-v)^2 + u^2 = 4((u-1)^2 + v^2)$ $3u^2 + 3v^2 - 8u + 4v = 0 \quad \text{oe}$ $\left(u - \frac{4}{3}\right)^2 + \left(v + \frac{2}{3}\right)^2 = \frac{20}{9} \quad \text{or} \quad u^2 + v^2 - \frac{8}{3}u + \frac{4}{3}v = 0$ <p>(i) centre is $\left(\frac{4}{3}, -\frac{2}{3}\right)$</p> <p>(ii) radius is $\frac{2\sqrt{5}}{3}$ oe</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>dM1</p> <p>A1</p> <p>A1 [8]</p>
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4 - (WFM01/F2(IAL)_Summer_2021_Q7) - Complex Numbers

<p>(a)</p> $(\cos \theta + i \sin \theta)^4 = \cos 4\theta + i \sin 4\theta$ $\cos^4 \theta + 4 \cos^3 \theta (i \sin \theta) + \frac{4 \times 3}{2!} \cos^2 \theta (i \sin \theta)^2$ $+ \frac{4 \times 3 \times 2}{3!} \cos \theta (i \sin \theta)^3 + (i \sin \theta)^4$ $= \cos^4 \theta + 4i \cos^3 \theta \sin \theta + i^2 6 \cos^2 \theta \sin^2 \theta + 4i^3 \cos \theta \sin^3 \theta + i^4 \sin^4 \theta$ $\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$ $\sin 4\theta = 4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta$ $\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4 \cos^3 \theta \sin \theta - 4 \cos \theta \sin^3 \theta}{\cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta}$ $\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} \quad *$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1A1* (6)</p>
<p>(b)</p> $x = \tan \theta \quad \frac{2 \tan \theta - 2 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta} = \frac{1}{2} \tan 4\theta = 1$ $\tan 4\theta = 2$ $x = \tan \theta = 0.284, 1.79$	<p>M1</p> <p>A1A1 (3)</p> <p>[9]</p>

5 - (WFM01/F2(IAL)_Winter_2021_Q1) - Complex Numbers

$i(1 + \sqrt{3}) = \frac{i(1 + \sqrt{3}) + pi}{i^2(1 + \sqrt{3}) + 3}$ $-i(1 + \sqrt{3})^2 + 3i(1 + \sqrt{3}) = i(1 + \sqrt{3}) + pi$ $-1 - 2\sqrt{3} - 3 + 3 + 3\sqrt{3} = 1 + \sqrt{3} + p$ $p = -2$	<p>M1</p> <p>dM1</p> <p>A1</p> <p>[3]</p>
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