A-Level Edexcel

FURTHER PURE MATHEMATICS

UNIT F2(IAL)

2020 — 2023

1 - (WFM01/F2(IAL)_Summer_2020_Q4) - *Complex Numbers*

(a) Express the complex number $18\sqrt{3} - 18\mathrm{i}$ in the form

$$r(\cos\theta + \mathrm{i}\sin\theta) \qquad -\pi < \theta \leqslant \pi$$

(3)

(b) Solve the equation

$$z^4 = 18\sqrt{3} - 18\mathrm{i}$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $-\pi < \theta \leqslant \pi$

(5)

2 - (WFM01/F2(IAL)_Summer_2020_Q5) - *Complex Numbers*

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - 3\mathrm{i}}{z + 2\mathrm{i}} \qquad z \neq -2\mathrm{i}$$

The circle with equation $|z| = 1$ in the z-plane is mapped by T onto the circle C in the w-plane.

Determine

(i) the centre of C,

(ii) the radius of C.

(7)

3 - (WFM01/F2(IAL)_Summer_2021_Q2) - *Complex Numbers*

The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i}$$

The transformation T maps the circle $|z| = 2$ in the z-plane onto a circle C in the w-plane.

Find (i) the centre of C,

(ii) the radius of C.

(8)

4 - (WFM01/F2(IAL)_Summer_2021_Q7) - *Complex Numbers*

(a) Use de Moivre's theorem to show that

$$\tan 4\theta \equiv \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

(6)

(b) Use the identity given in part (a) to find the 2 positive roots of

$$x^4 + 2x^3 - 6x^2 - 2x + 1 = 0$$

giving your answers to 3 significant figures.

(3)

5 - (WFM01/F2(IAL)_Winter_2021_Q1) - *Complex Numbers*

The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z + p\mathrm{i}}{\mathrm{i}z + 3} \qquad z \neq 3\mathrm{i} \quad p \in \mathbb{Z}$$

The point representing $\mathrm{i}\left(1 + \sqrt{3}\right)$ is invariant under T.

Determine the value of p.

(3)

6 - (WFM01/F2(IAL)_Winter_2021_Q8) - *Complex Numbers*

Given that $z = e^{i\theta}$

(a) show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0 \qquad 0 \leqslant \theta \leqslant \pi$$

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} \left(32\cos^6\theta - 4\cos^2\theta\right) d\theta$$

Solutions relying entirely on calculator technology are not acceptable.

(5)

ANSWERS

1 - (WFM01/F2(IAL)_Summer_2020_Q4) - *Complex Numbers*

(a)	$\left\|18\sqrt{3}-18\mathrm{i}\right\| = 18\sqrt{(3+1)} = 36$	B1
	$\tan\theta = \frac{-18}{18\sqrt{3}} \quad \theta = -\frac{\pi}{6}, \quad 18\sqrt{3}-18\mathrm{i} = 36\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right)$	M1,A1cao (3)
(b)	$z^4 = 36\left(\cos -\frac{\pi}{6}+\mathrm{i}\sin -\frac{\pi}{6}\right) = 36\left(\cos\left(2k\pi-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(2k\pi-\frac{\pi}{6}\right)\right)$	M1
	$z = \sqrt{6}\left(\cos\left(\frac{12k\pi-\pi}{24}\right)+\mathrm{i}\sin\left(\frac{12k\pi-\pi}{24}\right)\right)$	M1
	$k=0 \quad z_0 = \sqrt{6}\left(\cos\left(\frac{-\pi}{24}\right)+\mathrm{i}\sin\left(\frac{-\pi}{24}\right)\right) = \sqrt{6}\,\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{24}\right)}$	B1
	$k=1 \quad z_1 = \sqrt{6}\left(\cos\left(\frac{11\pi}{24}\right)+\mathrm{i}\sin\left(\frac{11\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\frac{11\pi}{24}}$	A1ft
	$k=2 \quad z_2 = \sqrt{6}\left(\cos\left(\frac{23\pi}{24}\right)+\mathrm{i}\sin\left(\frac{23\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\frac{23\pi}{24}}$	
	$k=-1 \quad z_3 = \sqrt{6}\left(\cos\left(-\frac{13\pi}{24}\right)+\mathrm{i}\sin\left(-\frac{13\pi}{24}\right)\right) = \sqrt{6}\mathrm{e}^{\mathrm{i}\left(-\frac{13\pi}{24}\right)}$	A1ft (5)
		[8]

2 - (WFM01/F2(IAL)_Summer_2020_Q5) - *Complex Numbers*

	$w = \frac{z-3\text{i}}{z+2\text{i}}$	
	$w(z+2\text{i}) = z - 3\text{i} \quad z = \frac{\text{i}(2w+3)}{1-w}$	M1
	$\lvert z\rvert = 1 \quad \left\lvert\frac{\text{i}(2w+3)}{1-w}\right\rvert = 1$	dM1
	$\lvert \text{i}(2w+3)\rvert = \lvert 1-w\rvert$	
	$w = u + \text{i}v \quad (2u+3)^2 + 4v^2 = (1-u)^2 + v^2$	ddM1
	$4u^2 + 12u + 9 + 4v^2 = 1 - 2u + u^2 + v^2$	
	$3u^2 + 3v^2 + 14u + 8 = 0$	dddM1
	$u^2 + v^2 + \frac{14}{3}u + \frac{8}{3} = 0$	A1
	$\left(u + \frac{7}{3}\right)^2 + v^2 = -\frac{8}{3} + \frac{49}{9} = \frac{25}{9}$	
(i)	Centre $\left(-\frac{7}{3}, 0\right)$	A1
(ii)	Radius $\frac{5}{3}$	A1 (7)
		[7]

3 - (WFM01/F2(IAL)_Summer_2021_Q2) - *Complex Numbers*

	$w = \dfrac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i}$	
	$z = \dfrac{2+\mathrm{i}w}{w-1}$	M1
	$\lvert z\rvert = 2 \Rightarrow \left\lvert \dfrac{2+\mathrm{i}w}{w-1} \right\rvert = 2 \Rightarrow \lvert 2+\mathrm{i}w\rvert = 2\lvert w-1\rvert$	
	$\lvert 2+\mathrm{i}u-v\rvert = 2\lvert u+\mathrm{i}v-1\rvert$	M1 A1
	$(2-v)^2 + u^2 = 4\left((u-1)^2+v^2\right)$	M1 A1
	$3u^2+3v^2-8u+4v=0$ oe	
	$\left(u-\dfrac{4}{3}\right)^2+\left(v+\dfrac{2}{3}\right)^2=\dfrac{20}{9}$ or $u^2+v^2-\dfrac{8}{3}u+\dfrac{4}{3}v=0$	dM1
	(i) centre is $\left(\dfrac{4}{3},-\dfrac{2}{3}\right)$	A1
	(ii) radius is $\dfrac{2\sqrt{5}}{3}$ oe	A1 [8]

4 - (WFM01/F2(IAL)_Summer_2021_Q7) - *Complex Numbers*

(a)	$(\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \mathrm{i}\sin 4\theta$	
	$\cos^4\theta + 4\cos^3\theta(\mathrm{i}\sin\theta) + \frac{4\times 3}{2!}\cos^2\theta(\mathrm{i}\sin\theta)^2 + \frac{4\times 3\times 2}{3!}\cos\theta(\mathrm{i}\sin\theta)^3 + (\mathrm{i}\sin\theta)^4$	M1
	$= \cos^4\theta + 4\mathrm{i}\cos^3\theta\sin\theta + \mathrm{i}^2 6\cos^2\theta\sin^2\theta + 4\mathrm{i}^3\cos\theta\sin^3\theta + \mathrm{i}^4\sin^4\theta$	A1
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	M1
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	A1
	$\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$	
	$\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ *	M1A1* (6)
(b)	$x = \tan\theta \qquad \frac{2\tan\theta - 2\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = \frac{1}{2}\tan 4\theta = 1$	
	$\tan 4\theta = 2$	M1
	$x = \tan\theta = 0.284,\ 1.79$	A1A1 (3) [9]

5 - (WFM01/F2(IAL)_Winter_2021_Q1) - *Complex Numbers*

	$\mathrm{i}(1+\sqrt{3}) = \frac{\mathrm{i}(1+\sqrt{3}) + p\mathrm{i}}{\mathrm{i}^2(1+\sqrt{3}) + 3}$	M1
	$-\mathrm{i}(1+\sqrt{3})^2 + 3\mathrm{i}(1+\sqrt{3}) = \mathrm{i}(1+\sqrt{3}) + p\mathrm{i}$	
	$-1 - 2\sqrt{3} - 3 + 3 + 3\sqrt{3} = 1 + \sqrt{3} + p$	dM1
	$p = -2$	A1 **[3]**