PURE MATHEMATICS

UNIT P4(IAL)

2019 — 2023

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1 - (WMA11/P4(IAL)_Winter_2021_Q4) - *Algebra And Functions*

The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1-2t}{3+t} \qquad t > 0$$

(a) Show that the equation of C can be written in the form $y = \mathrm{g}(x)$ where g is the function

$$\mathrm{g}(x) = \frac{ax+b}{cx+d} \qquad x > k$$

where a, b, c, d and k are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g.

(2)

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1 - (WMA11/P4(IAL)_Winter_2022_Q3) - *Coordinate Geometry In The (x, Y) Plane*

The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

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2 - (WMA11/P4(IAL)_Winter_2023_Q2) - *Coordinate Geometry In The (x, Y) Plane*

A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

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1 - (WMA11/P4(IAL)_Summer_2020_Q4) - *Differentiation*

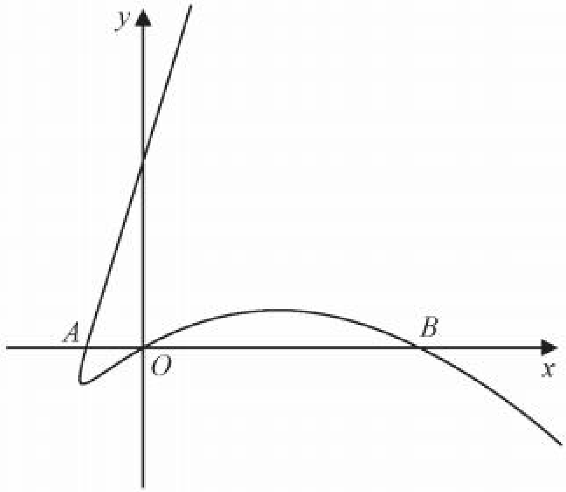

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

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2 - (WMA11/P4(IAL)_Summer_2020_Q6) - *Differentiation*

A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x \ln x = 0$$

(2)

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3 - (WMA11/P4(IAL)_Summer_2021_Q3) - *Differentiation*

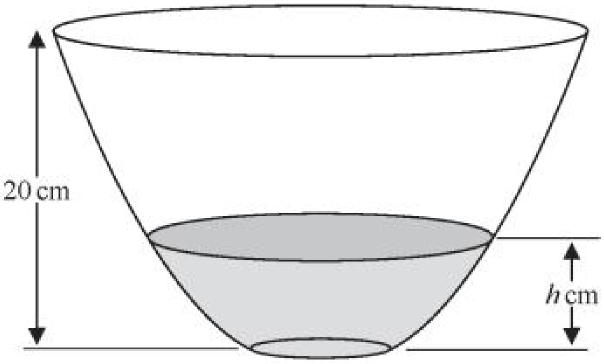

Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h+4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

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4 - (WMA11/P4(IAL)_Summer_2021_Q5) - *Differentiation*

A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

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5 - (WMA11/P4(IAL)_Winter_2021_Q6) - *Differentiation*

A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

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6 - (WMA11/P4(IAL)_Summer_2022_Q3) - *Differentiation*

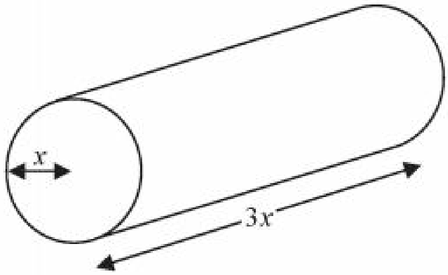

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\frac{dx}{dt}$ when $x = 7$

(4)

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

(4)

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ANSWERS

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1 - (WMA11/P4(IAL)_Winter_2021_Q4) - *Algebra And Functions*

(a)	$k = 2$ or $x > 2$	B1
	$t = \dfrac{1}{x-2} \Rightarrow y = \dfrac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}}$	M1 A1
	$\dfrac{1-\frac{2}{x-2}}{3+\frac{1}{x-2}} = \dfrac{x-2-2}{\ldots}$ or $\dfrac{\ldots}{3(x-2)+1}$	A1 (M1 on EPEN)
	$y = \dfrac{x-4}{3x-5}$	A1
		(5)
(b)	$-2 < \mathrm{g} < \dfrac{1}{3}$	M1 A1
		(2)
		(7 marks)

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1 - (WMA11/P4(IAL)_Winter_2022_Q3) - *Coordinate Geometry In The (x, Y) Plane*

(a)	$x = 3 + 2\sin t \qquad y = \dfrac{6}{7 + \cos 2t} \qquad -\dfrac{\pi}{2} \leqslant t \leqslant \dfrac{\pi}{2}$	
	$y = \dfrac{6}{7 + 1 - 2\sin^2 t}$	**M1**
	$\sin t = \dfrac{x-3}{2} \Rightarrow y = \dfrac{6}{8 - 2\left(\dfrac{x-3}{2}\right)^2}$	**M1A1**
	$\Rightarrow y = \dfrac{12}{16 - (x-3)^2} = \dfrac{12}{(4 - x + 3)(4 + x - 3)} = \dfrac{12}{(7-x)(1+x)}$ *	**M1A1***
	$\left(t = -\dfrac{\pi}{2} \Rightarrow x = 1, t = \dfrac{\pi}{2} \Rightarrow x = 5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
		(6)
(b)	$\dfrac{12}{(7-x)(1+x)} = \dfrac{A}{7-x} + \dfrac{B}{1+x} \Rightarrow 12 = A(x+1) + B(7-x) \Rightarrow A = ..., B = ...$	**M1**
	So $(y =)\dfrac{3}{2(x+1)} - \dfrac{3}{2(x-7)}$ oe	**A1A1**
		(3)
		(9 marks)

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2 - (WMA11/P4(IAL)_Winter_2023_Q2) - *Coordinate Geometry In The (x, Y) Plane*

(a)	E.g. $x=\frac{t-1}{2t+1}\Rightarrow t=\frac{x+1}{1-2x}$ or $y=\frac{6}{2t+1}\Rightarrow t=\frac{6-y}{2y}$	**M1**
	E.g. $y=\frac{6}{2t+1}\Rightarrow y=\frac{6}{2\times\left(\frac{x+1}{1-2x}\right)+1}$ or $t=\frac{6-y}{2y}\Rightarrow x=\frac{\frac{6-y}{2y}-1}{2\times\frac{6-y}{2y}+1}$	**A1**
	E.g. $y=\frac{6}{2\times\left(\frac{x+1}{1-2x}\right)+1}\Rightarrow y=\frac{6(1-2x)}{2\times(x+1)+1(1-2x)}=ax+b$	**dM1**
	E.g. $y=\frac{6(1-2x)}{3}, y=2(1-2x)$ oe so linear *	**A1***
		(4)
(b)	$y=2(1-2x)$ and $y=x+12\Rightarrow 2(1-2x)=x+12\Rightarrow x=\ldots$	**M1**
	$x=-2$	**A1cao**
		(2)
Alt (b)	$\frac{6}{2t+1}=\frac{t-1}{2t+1}+12\Rightarrow t=\left(-\frac{1}{5}\right)$	**M1**
	$x=\frac{-\frac{1}{5}-1}{2\times-\frac{1}{5}+1}=-2$	**A1**
		(2)
		(6 marks)

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1 - (WMA11/P4(IAL)_Summer_2020_Q4) - *Differentiation*

(a)	A and B are where $t^3-4t=0 \Rightarrow t(t^2-4)=0 \Rightarrow t=2$ or -2	M1
	Substitutes $t=2, x=2\times4-6\times2=-4$ Hence $A=(-4,0)$	A1
	When $t=-2, x=2\times4-6\times-2=20, (y=0)$ Hence $B=(20,0)$ *	B1*
		(3)
(b)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3t^2-4}{4t-6}$	M1A1
	Sub $t=-2$ into $\frac{dy}{dx}=\frac{3t^2-4}{4t-6} \Rightarrow$ gradient $=\left(-\frac{4}{7}\right)$	M1
	Uses their $\left(-\frac{4}{7}\right)$ and $(20,0)$ to produce eqn of tangent $\Rightarrow 7y+4x-80=0$ *	M1 A1*
		(5)
(c)	Substitutes $x=2t^2-6t, y=t^3-4t$, into $7y+4x-80=0$	
	$\Rightarrow 7(t^3-4t)+4(2t^2-6t)-80=0$	M1
	$\Rightarrow 7t^3+8t^2-52t-80=0$	A1
	$\Rightarrow (t+2)^2(7t-20)=0$	
	$t=\text{"}\frac{20}{7}\text{"} \Rightarrow x=....$	dM1
	$x=-\frac{40}{49}$	A1
		(4)
		(12 marks)

2 - (WMA11/P4(IAL)_Summer_2020_Q6) - *Differentiation*

(a)	$y=x^{\sin x} \Rightarrow \ln y=\sin x\ln x$	B1
	Differentiates $\frac{1}{y}\frac{dy}{dx}=\frac{\sin x}{x}+\ln x\cos x$	M1 M1 A1
	$\frac{dy}{dx}=\frac{y\sin x}{x}+y\ln x\cos x$ oe	A1
		(5)
(b)	Puts $\frac{dy}{dx}=0 \Rightarrow \frac{\sin x}{x}+\ln x\cos x=0$	M1
	$\frac{\sin x}{\cos x}+x\ln x=0 \Rightarrow \tan x+x\ln x=0$ *	A1*
		(2)
		(7 marks)

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3 - (WMA11/P4(IAL)_Summer_2021_Q3) - *Differentiation*

(a)	Attempts to find $\frac{\frac{1}{3}\times 20^2 \times 24}{160} = 20$ seconds	M1 A1
		(2)
(b)	Attempts $\frac{dV}{dh} = h^2 + \frac{8}{3}h$	M1
	Attempts to use $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 160 = \left(h^2 + \frac{8}{3}h\right) \times \frac{dh}{dt}$	M1 A1
	Substitutes $h = 5 \Rightarrow 160 = \left(5^2 + \frac{40}{3}\right) \times \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{96}{23}$ (4.2) cm s^{-1}	dM1 A1
		(5)
		(7 marks)

4 - (WMA11/P4(IAL)_Summer_2021_Q5) - *Differentiation*

(a)	$2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$	B1 M1 A1
	$\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$ *	A1*
		(4)
(b)	Puts $x = 0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$	B1
	Attempts tangent at $(0,0)$ **or** $(0,1)$ $y = 3x$ **or** $y = -5x + 1$	M1 A1
	Solves $y = 3x$ with $y = -5x + 1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	dM1 A1
		(5)
		(9 marks)

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5 - (WMA11/P4(IAL)_Winter_2021_Q6) - *Differentiation*

(a)	$4y^2+3x=6y\,e^{-2x}$	
	$4y^2+3x \to 8y\dfrac{dy}{dx}+3$	B1
	$6y\,e^{-2x} \to -12ye^{-2x}+6e^{-2x}\dfrac{dy}{dx}$	M1 A1
	$8y\dfrac{dy}{dx}+3=-12ye^{-2x}+6e^{-2x}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx}=\dfrac{12ye^{-2x}+3}{6e^{-2x}-8y}$ oe	M1 A1
		(5)
(b)	Sets $x=0$ in $4y^2+3x=6y\,e^{-2x} \Rightarrow y=\dfrac{3}{2}$ oe	B1
	Substitutes $\left(0,\dfrac{3}{2}\right)$ in their $\dfrac{dy}{dx}=\dfrac{12ye^{-2x}+3}{6e^{-2x}-8y}=\left(\dfrac{7}{-2}\right)$	M1
	$m_N=-1\div"\dfrac{7}{-2}" \Rightarrow y="\dfrac{2}{7}"x+"\dfrac{3}{2}"$	**d**M1
	$y=\dfrac{2}{7}x+\dfrac{3}{2}$ oe e.g. $y=\dfrac{6}{21}x+\dfrac{3}{2}$	A1
		(4)
		(9 marks)

6 - (WMA11/P4(IAL)_Summer_2022_Q3) - *Differentiation*

(a)	$\dfrac{dA}{dt}=-0.5$	B1
	$A=\pi x^2 \Rightarrow \dfrac{dA}{dx}=2\pi x$	B1
	$\dfrac{dx}{dt}=\dfrac{dA}{dt}\div\dfrac{dA}{dx}=\dfrac{"-0.5"}{"2\pi x"} \quad \left(=\dfrac{-1}{4\pi x}\right)$	M1
	$\dfrac{dx}{dt}=-0.011368...$	A1cso
		(4)
(b)	$V=\pi x^2(3x)=3\pi x^3$	B1
	$\dfrac{dV}{dx}=9\pi x^2$	B1ft
	$\dfrac{dV}{dt}=\dfrac{dV}{dx}\times\dfrac{dx}{dt}=9\pi x^2\times"-\dfrac{1}{4\pi x}" \quad (=-2.25x)$	M1
	$\left(\dfrac{dV}{dt}=\right)-9 \Rightarrow$ (Rate of decrease $=$) 9 $\left(mm^3\ s^{-1}\right)$	A1
		(4)
		(8 marks)