PURE MATHEMATICS

UNIT P3(IAL)

2019 — 2023

1 – (WMA11/P3(IAL)_Summer_2020_Q3) – *Algebra And Functions*

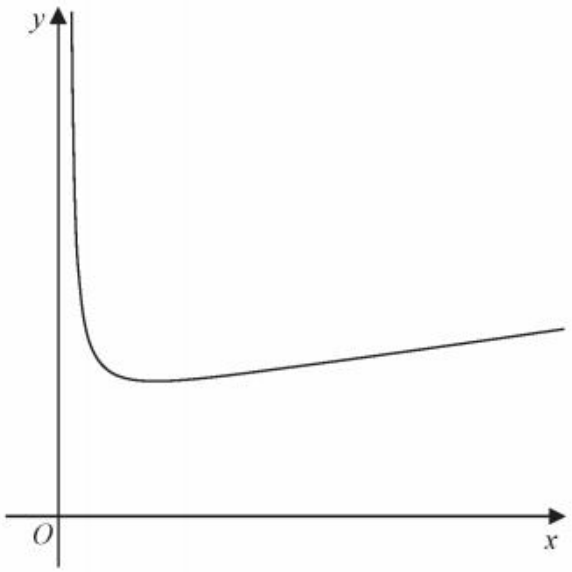

Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x + 3}{\sqrt{4x - 1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$.

(4)

(b) Hence find the range of f.

(3)

2 - (WMA11/P3(IAL)_Summer_2020_Q4) - *Algebra And Functions*

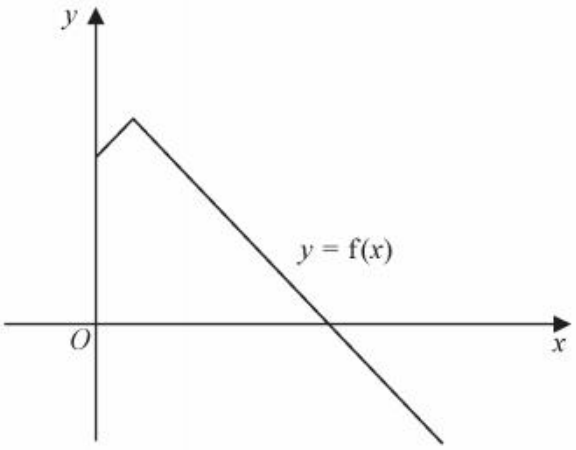

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find $\mathrm{ff}(6)$

(2)

(b) Solve the equation $\mathrm{f}(x) = 5x$

(2)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k.

(2)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x - b)$

The vertex of the graph with equation $y = a\mathrm{f}(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

3 - (WMA11/P3(IAL)_Winter_2020_Q1) - *Algebra And Functions*

A population of a rare species of toad is being studied.

The number of toads, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \qquad t \geqslant 0, t \in \mathbb{R}$$

According to this model,

(a) calculate the number of toads in the population at the start of the study,

(1)

(b) find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places.

(4)

(c) Explain why, according to this model, the number of toads in the population can never reach 500

(1)

4 - (WMA11/P3(IAL)_Winter_2020_Q2) - *Algebra And Functions*

The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$

(2)

(b) Find f^{-1}

(3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

(3)

5 - (WMA11/P3(IAL)_Winter_2020_Q4) - *Algebra And Functions, Trigonometry*

(i) $$f(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $f'(x)$ in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(6)

(ii) $$g(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = g(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

6 - (WMA11/P3(IAL)_Winter_2020_Q6) - *Algebra And Functions*

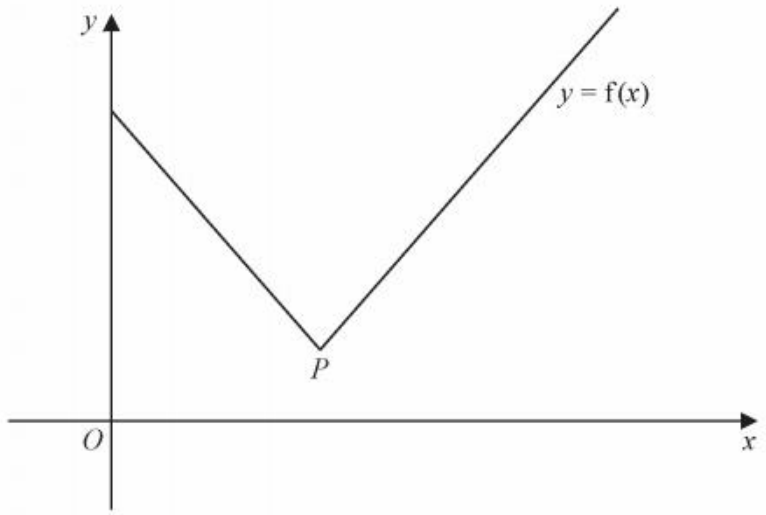

Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P.

(2)

(b) Solve the equation $f(x) = 3x - 2$

(4)

Given that the equation

$$f(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k.

(3)

ANSWERS

1 - (WMA11/P3(IAL)_Summer_2020_Q3) - *Algebra And Functions*

(a)	$\frac{dy}{dx}=\frac{(4x-1)^{\frac{1}{2}}\times 2-(2x+3)\times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}$	M1 A1
	$\frac{(4x-1)^{\frac{1}{2}}\times 2-(2x+3)\times 2(4x-1)^{-\frac{1}{2}}}{(4x-1)}\times\frac{(4x-1)^{\frac{1}{2}}}{(4x-1)^{\frac{1}{2}}}=\frac{4x-8}{(4x-1)^{\frac{3}{2}}}$	dM1 A1
		(4)
(b)	Turning point where $\frac{dy}{dx}=0\Rightarrow x=2$	M1
	Find value of f at $x=2\Rightarrow f(x)=\sqrt{7}$ Hence range is $f\geqslant\sqrt{7}$	dM1 A1
		(3)
		(7 marks)

2 - (WMA11/P3(IAL)_Summer_2020_Q4) - *Algebra And Functions*

(a)	$ff(6)=f(13)=-1$	M1 A1
		(2)
(b)	Attempts $21+2(2-x)=5x\Rightarrow x=...$ or $21-2(x-2)=5x\Rightarrow x=...$	M1
	$x=\frac{25}{7}$ only	A1
		(2)
(c)	Either $k<21$ or $k\geqslant 17$	M1
	$17\leqslant k<21$	A1
		(2)
(d)	$a=\frac{1}{7}$ $b=4$	B1 B1
		(2)
		(8 marks)

3 - (WMA11/P3(IAL)_Winter_2020_Q1) - *Algebra And Functions*

(a)	$P_0=300$	B1
		(1)
(b)	$420=\frac{900e^{0.12t}}{2e^{0.12t}+1}\Rightarrow 60e^{0.12t}=420$	M1 A1
	Correct use of lns $\Rightarrow t=\frac{\ln 7}{0.12}=16.22$	dM1 A1
		(4)
(c)	States that maximum number (upper limit) is 450 so cannot reach 500	B1
		(1)
		6 marks

4 - (WMA11/P3(IAL)_Winter_2020_Q2) - *Algebra And Functions*

(a)	$\text{fg}\left(e^2\right)=\text{f}\left(\frac{5}{2}\ln e^2\right)=\frac{12}{\frac{5}{2}\ln e^2+1},=2$	M1, A1 (2)
(b)	$\text{f}(x)=\frac{12}{x+1}$ $\text{f}^{-1}(x)=\frac{12}{x}-1$ $0<x<12$	M1 A1 B1 (3)
(c)	$\frac{12}{x+1}=\frac{12}{x}-1\Rightarrow 12x=12(x+1)-x(x+1)$ $\Rightarrow x^2+x-12=0\Rightarrow x=\ldots$ Must be 3TQ $x=3$ only	M1 dM1 A1 (3) **8 marks**
(c) Alts	Solves $\text{f}^{-1}(x)=x\Rightarrow\frac{12}{x}-1=x$ leading to quadratic equation, or solves $\text{f}(x)=x\Rightarrow\frac{12}{x+1}=x$ leading to quadratic equation $\Rightarrow x^2+x-12=0\Rightarrow x=\ldots$ Must be 3TQ $x=3$ only	M1 dM1 A1 **(3)**

5 - (WMA11/P3(IAL)_Winter_2020_Q4) - *Algebra And Functions, Trigonometry*

(i) (a)	$\text{f}'(x)=\frac{4(x-3)(2x+5)-(2x+5)^2}{(x-3)^2}$ or $\frac{(x-3)(8x+20)-(4x^2+20x+25)}{(x-3)^2}$ $=\frac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1 M1 A1
(b)	Attempts both critical values or finds one "correct" end $x<-2.5,\ x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$)	M1 A1 (6)
(ii)	Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}}\rightarrow A(\sin 4x)^{-\frac{1}{2}}\times\cos 4x$	M1
	$\text{g}(x)=x(\sin 4x)^{\frac{1}{2}}\Rightarrow \text{g}'(x)=(\sin 4x)^{\frac{1}{2}}+x\times\frac{1}{2}(\sin 4x)^{-\frac{1}{2}}4\cos 4x$	M1 A1
	Sets $\text{g}'(x)=0\rightarrow(\sin 4x)^{\frac{1}{2}}+x\times\frac{2\cos 4x}{(\sin 4x)^{\frac{1}{2}}}=0$ and $\times\frac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}$ oe	M1
	$\rightarrow\tan 4x+2x=0$	A1 (5) **11 marks**

6 – (WMA11/P3(IAL)_Winter_2020_Q6) – *Algebra And Functions*

(a)	$(2.5, 3)$ oe	B1 B1
		(2)
(b)	Attempts one solution usually $4x-10+3=3x-2 \Rightarrow x=5$	M1 A1
	Attempts both solutions $-4x+10+3=3x-2 \Rightarrow x=\frac{15}{7}$	dM1 A1
		(4)
(c)	Attempts to solve $y=kx+2$ with $x=2.5, y=3$ or states that $k<4$	M1
	$k \ldots \frac{2}{5}$	A1
	States $\frac{2}{5}<k<4$	A1
		(3)
		9 marks

7 – (WMA11/P3(IAL)_Winter_2020_Q9) – *Trigonometry, Algebra And Functions*

(a)	$R=\sqrt{41}$	B1
	$\tan\alpha=\frac{4}{5} \Rightarrow \alpha=$ awrt 0.675	M1A1
		(3)
(b)	(i) Describes stretch: stretch in the y direction by "$\sqrt{41}$"	B1 ft
	(ii) Describes translation: E.g. translate by $\begin{pmatrix} -\arctan\frac{4}{5} \\ 0 \end{pmatrix}$	B1 ft
		(2)
(c)	Attempts either $g(\theta)=\frac{90}{4+\left(\sqrt{41}\right)^2}$ OR $g(\theta)=\frac{90}{4}$	M1
	Range $2 \leqslant g(\theta) \leqslant 22.5$	A1
		(2)
		7 marks