PURE MATHEMATICS

UNIT P2(IAL)

2019 — 2023

1 - (WMA11/P2(IAL)_Summer_2019_Q6) - *Algebra And Functions, Trigonometry*

$$f(x) = kx^3 - 15x^2 - 32x - 12 \qquad \text{where } k \text{ is a constant}$$

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$

(2)

(b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

(c) Solve, for $0 \leqslant \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place.

(2)

2 - (WMA11/P2(IAL)_Summer_2020_Q3) - *Algebra And Functions, Differentiation*

$$f(x) = ax^3 - x^2 + bx + 4$$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

3 - (WMA11/P2(IAL)_Winter_2020_Q3) - *Algebra And Functions, Trigonometry*

$$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.

(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.

(2)

4 - (WMA11/P2(IAL)_Winter_2021_Q1) - *Algebra And Functions*

$$f(x) = x^4 + ax^3 - 3x^2 + bx + 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

5 - (WMA11/P2(IAL)_Summer_2022_Q7) - *Algebra And Functions, Integration*

$$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

ANSWERS

1 - (WMA11/P2(IAL)_Summer_2019_Q6) - *Algebra And Functions, Trigonometry*

(a)	Sets $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$	M1
	$\Rightarrow 27k=243 \Rightarrow k=9$ * (= 0 must be seen)	A1*
		(2)
(b)	$9x^3-15x^2-32x-12=(x-3)(9x^2+12x+4)$	M1 A1
	$=(x-3)(3x+2)^2$	dM1 A1
		(4)
(c)	Attempts $\cos\theta=-\frac{2}{3}$	M1
	$\theta=131.8°, 228.2°$ (awrt)	A1
		(2)
		(8 marks)

2 - (WMA11/P2(IAL)_Summer_2020_Q3) - *Algebra And Functions, Differentiation*

(a)	$a(-4)^3-(-4)^2+b(-4)+4=-108$ Attempts to set $f(-4)=-108$ to obtain an equation in a and b. Score when you see "– 4" embedded in the equation or 2 correct terms (excluding the "+ 4") on lhs. May be implied by e.g. $-64a-16-4b+4=-108$ Condone minor slips on the lhs e.g. one sign error between terms but must use – 108		M1
	As an alternative for the first mark we will condone an attempt at long division. This requires a complete method to divide (ax^3-x^2+bx+4) by $(x+4)$ to obtain a remainder in terms of a and b which is then equated to -108 For reference, the quotient is $ax^2-(1+4a)x+16a+b+4$ and the remainder is $-4b-64a-12$		
	$-64a-16-4b+4=-108$ $\Rightarrow 16a+b=24$ *	Correct equation obtained with no errors and at least one line of intermediate working if starting with e.g. $a(-4)^3-(-4)^2+b(-4)+4=-108$	A1*
			(2)
(b)	$a\left(\frac{1}{2}\right)^3-\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)+4=0$ Attempts to set $f\left(\frac{1}{2}\right)=0$ to obtain an equation in a and b. Condone slips. Score when you see "$\frac{1}{2}$" embedded in the equation or 2 correct terms (excluding the "+ 4") on lhs. May be implied by e.g. $\frac{a}{8}-\frac{1}{4}+\frac{b}{2}+4=0$ The "= 0" may be implied when they attempt to solve simultaneously below		M1
	An alternative for the first mark is to attempt long division. This requires a complete method to divide (ax^3-x^2+bx+4) by $(2x-1)$ to obtain a remainder in a and b which is then equated to 0 For reference, the quotient is $\frac{a}{2}x^2+\left(\frac{a}{4}-\frac{1}{2}\right)x+\left(\frac{b}{2}-\frac{1}{4}+\frac{a}{8}\right)$ and the remainder is $\frac{15}{4}+\frac{b}{2}+\frac{a}{8}$		
	$16a+b=24,\ a+4b=-30$ $\Rightarrow a=...,b=...$	Attempts to solve $16a+b=24$ simultaneously with their equation in a and b. This may be implied if values of a and b are obtained (e.g. calculator)	M1
	$a=2,\ b=-8$	Correct values	A1
			(3)
(c)	$f(x)=2x^3-x^2-8x+4$ $\Rightarrow f'(x)=6x^2-2x-8$	Correct derivative (follow through their a and b). Allow unsimplified and apply isw if necessary. Allow with the letters "a" and "b" and a "made up" "a" and "b".	B1ft
			(1)

(d)	$6x^2-2x-8=0$ $\Rightarrow(3x-4)(x+1)=0$ $\Rightarrow x=\ldots$	Sets their $f'(x)=0$ (may be implied) and solves a 3 term quadratic. Apply general guidance if necessary. You may need to check if a calculator has been used.	M1
	$x=\frac{4}{3},\ -1\Rightarrow y=\ldots$	Uses at least one of their x values to find a value for y using their $f(x)$ where x is from an attempt to solve $f'(x)=0$. You may need to check their y values if working is not shown.	M1
	$\left(\frac{4}{3},-\frac{100}{27}\right)$ **or** $(-1,9)$ Or e.g. $x=\frac{4}{3},\ y=-\frac{100}{27}$ and $x=-1,\ y=9$ One correct point. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. **Depends on having scored both previous M marks.**		A1
	$\left(\frac{4}{3},-\frac{100}{27}\right)$ **and** $(-1,9)$ Or e.g. $x=\frac{4}{3},\ y=-\frac{100}{27}$ and $x=-1,\ y=9$ Both correct points. The fractional coordinates must be exact but allow 1.3 with a dot over the 3 and 3.703 with dots over the 7 and 3. Note that it is not necessary for the points to be written as coordinates as long as the pairing is clear. **Depends on having scored both previous M marks.**		A1
	Fully correct answers with no working scores 4/4 following a correct part (c) i.e. $\Rightarrow f'(x)=6x^2-2x-8$		
			(4)
			Total 10

3 – (WMA11/P2(IAL)_Winter_2020_Q3) – *Algebra And Functions, Trigonometry*

(a)	Attempts $f\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+17\left(-\frac{3}{2}\right)^2+4\left(-\frac{3}{2}\right)-12$	M1
	$=0\Rightarrow(2x+3)$ is a factor *	A1*
		(2)
(b)	$6x^3+17x^2+4x-12=(2x+3)(3x^2+4x-4)$	M1 A1
	$=(2x+3)(3x-2)(x+2)$	dM1 A1
		(4)
(c)	Solves $\tan\theta=-\frac{3}{2}$ or "-2" or "$\frac{2}{3}$"	M1
	$\theta=$ awrt 2.03, 2.16	A1
		(2)
		(8 marks)

4 - (WMA11/P2(IAL)_Winter_2021_Q1) - *Algebra And Functions*

(a)	$f(-1)=(-1)^4+a(-1)^3-3(-1)^2+b(-1)+5=4$	M1
	$1-a-3-b+5=4 \Rightarrow a+b=-1$ *	A1*
		(2)
(b)	$f(2)=(2)^4+a(2)^3-3(2)^2+b(2)+5=-23$	M1
	$\Rightarrow 8a+2b=-32$ oe (eg $4a+b=-16$)	A1
	$b=-1-a \Rightarrow 4a-1-a=-16 \Rightarrow a=...$	dM1
	$a=-5, b=4$	A1
		(4)
		(6 marks)

5 - (WMA11/P2(IAL)_Summer_2022_Q7) - *Algebra And Functions, Integration*

	$(x+2)$ a factor $\Rightarrow f(-2)=0 \Rightarrow -8A+24+8+B=0$	**M1A1**
	$\int f(x)dx=\frac{A}{4}x^4+2x^3-2x^2+Bx$	**M1** **A1**
	$\int_3^5 f(x)dx=176 \Rightarrow \left[\frac{A}{4}x^4+2x^3-2x^2+Bx\right]_3^5=176$ $\Rightarrow \left(\frac{A}{4}5^4+2(5^3)-2(5^2)+5B\right)-\left(\frac{A}{4}3^4+2(3^3)-2(3^2)+3B\right)=176$	**dM1**
	$\left.\begin{matrix}8A-B=32\\136A+2B=12\end{matrix}\right\} \Rightarrow A=..., B=...$	**dM1**
	$A=\frac{1}{2}, B=-28$	**A1**
		(7)
		(7 marks)