

PURE MATHEMATICS

UNIT P1(IAL)

2019 — 2023

Chapter 1	Algebra And Functions	Page 1
Chapter 2	Coordinate Geometry In The (X, Y) Plane	Page 33
Chapter 3	Trigonometry	Page 48
Chapter 4	Differentiation	Page 65
Chapter 5	Integration	Page 81
Chapter 6	Proof	-----
Chapter 7	Sequences And Series	-----
Chapter 8	Exponentials And Logarithms	Page 96
Chapter 9	Numerical Methods	-----
Chapter 10	Binomial Expansion	-----
Chapter 11	Vectors	-----
	ANSWERS	Page 100

1 - (WMA11/P1(IAL)_Summer_2019_Q2) - Algebra And Functions

Answer this question showing each stage of your working.

(a) Simplify $\frac{1}{4 - 2\sqrt{2}}$

giving your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers.

(2)

(b) Hence, or otherwise, solve the equation

$$4x = 2\sqrt{2}x + 20\sqrt{2}$$

giving your answer in the form $p + q\sqrt{2}$ where p and q are rational numbers.

(3)

2 - (WMA11/P1(IAL)_Summer_2019_Q3) - Algebra And Functions

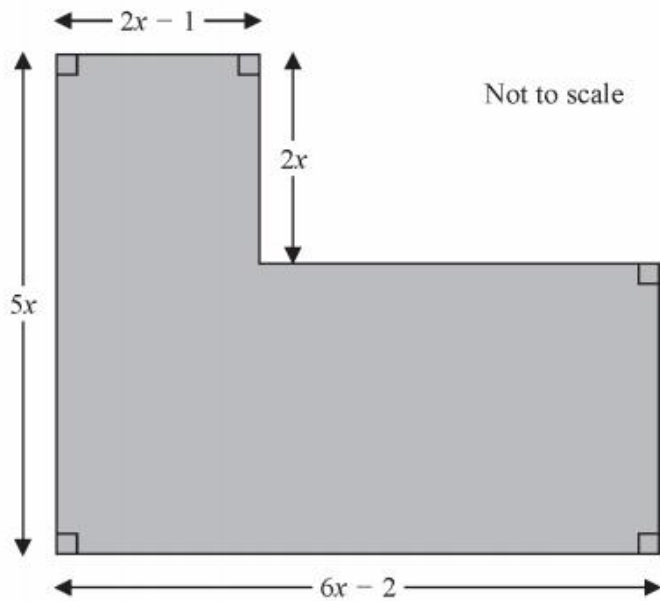


Figure 1

Figure 1 shows the plan of a garden. The marked angles are right angles.

The six edges are straight lines.

The lengths shown in the diagram are given in metres.

Given that the perimeter of the garden is greater than 29 m,

(a) show that $x > 1.5$ m

(3)

Given also that the area of the garden is less than 72 m^2 ,

(b) form and solve a quadratic inequality in x .

(5)

(c) Hence state the range of possible values of x .

(1)

3 - (WMA11/P1(IAL)_Summer_2019_Q5) - Algebra And Functions

(a) Find, using algebra, all real solutions of

$$2x^3 + 3x^2 - 35x = 0 \quad (3)$$

(b) Hence find all real solutions of

$$2(y - 5)^6 + 3(y - 5)^4 - 35(y - 5)^2 = 0 \quad (4)$$

4 - (WMA11/P1(IAL)_Summer_2019_Q6) - Algebra And Functions

The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(a) Find the value of c .

(4)

(b) Hence find the coordinates of the point of intersection.

(3)

www.exam-mate.com

5 - (WMA11/P1(IAL)_Winter_2019_Q4) - Algebra And Functions

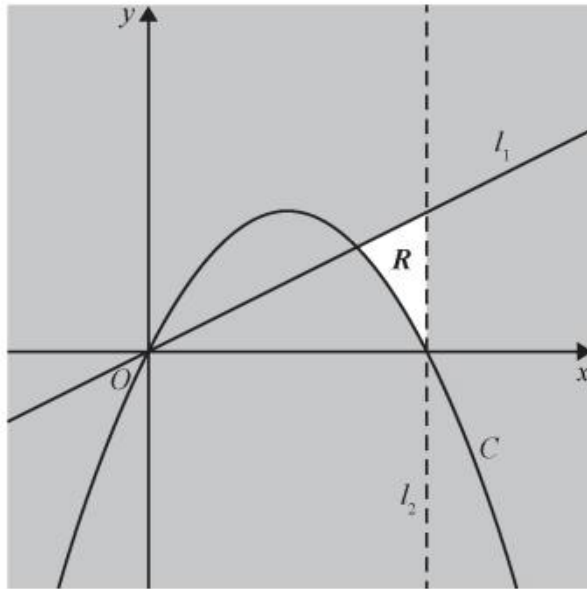
**Figure 1**

Figure 1 shows a line l_1 with equation $2y = x$ and a curve C with equation $y = 2x - \frac{1}{8}x^2$

The region R , shown unshaded in Figure 1, is bounded by the line l_1 , the curve C and a line l_2

Given that l_2 is parallel to the y -axis and passes through the intercept of C with the positive x -axis, identify the inequalities that define R .

(3)

6 - (WMA11/P1(IAL)_Winter_2019_Q8) - Algebra And Functions

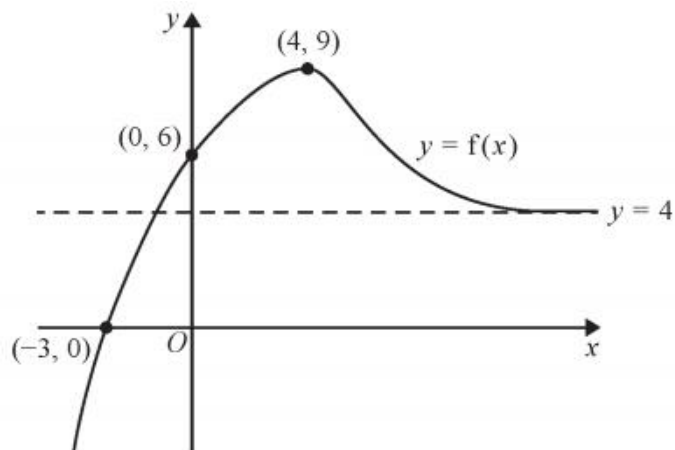


Figure 4

The curve C with equation $y = f(x)$ is shown in Figure 4.

The curve C

- has a single turning point, a maximum at $(4, 9)$
- crosses the coordinate axes at only two places, $(-3, 0)$ and $(0, 6)$
- has a single asymptote with equation $y = 4$

as shown in Figure 4.

(a) State the equation of the asymptote to the curve with equation $y = f(-x)$. (1)

(b) State the coordinates of the turning point on the curve with equation $y = f\left(\frac{1}{4}x\right)$. (1)

Given that the line with equation $y = k$, where k is a constant, intersects C at exactly one point,

(c) state the possible values for k . (2)

The curve C is transformed to a new curve that passes through the origin.

(d) (i) Given that the new curve has equation $y = f(x) - a$, state the value of the constant a . (1)

(ii) Write down an equation for another single transformation of C that also passes through the origin. (2)

7 - (WMA11/P1(IAL)_Winter_2019_Q9) - Algebra And Functions

The equation

$$\frac{3}{x} + 5 = -2x + c$$

where c is a constant, has no real roots.

Find the range of possible values of c .

(7)

www.exam-mate.com

ANSWERS

www.exam-mate.com

1 - (WMA11/P1(IAL)_Summer_2019_Q2) - Algebra And Functions

(a)	$\frac{1}{4-2\sqrt{2}} = \frac{1}{4-2\sqrt{2}} \times \frac{4+2\sqrt{2}}{4+2\sqrt{2}}$ $= \frac{4+2\sqrt{2}}{16-8} = \frac{1}{2} + \frac{1}{4}\sqrt{2} \quad \text{oe}$	M1 A1	(2)
(b)	$4x = 2\sqrt{2}x + 20\sqrt{2} \Rightarrow (4-2\sqrt{2})x = 20\sqrt{2}$ $\Rightarrow x = \frac{20\sqrt{2}}{(4-2\sqrt{2})} = 20\sqrt{2} \times \text{(a)}$ $\Rightarrow x = 20\sqrt{2} \times \left(\frac{1}{2} + \frac{1}{4}\sqrt{2}\right) = 10 + 10\sqrt{2}$	M1 dM1 A1	
			(5 marks)

2 - (WMA11/P1(IAL)_Summer_2019_Q3) - Algebra And Functions

(a)	<p>Attempts perimeter of garden = $2 \times 5x + 2 \times (6x - 2)$</p> <p>Sets $2 \times 5x + 2 \times (6x - 2) > 29 \Rightarrow 22x > 33$</p> $\Rightarrow x > \frac{33}{22} \Rightarrow x > 1.5 \quad *$	M1 dM1 A1*	(3)
(b)	<p>Attempts area of garden = $2x(2x-1) + 3x(6x-2)$</p> <p>Sets $A < 72 \Rightarrow 22x^2 - 8x - 72 < 0$</p> <p>Finds critical values $11x^2 - 4x - 36 \Rightarrow x = -\frac{18}{11}, 2$</p> <p>Chooses inside region</p> $-\frac{18}{11} < x < 2$	M1 A1 M1 ddM1 A1	
(c)	$1.5 < x < 2$	B1	(1)
			(9 marks)

3 - (WMA11/P1(IAL)_Summer_2019_Q5) - Algebra And Functions

(a)	$2x^3 + 3x^2 - 35x = 0 \Rightarrow x(2x^2 + 3x - 35) = 0$ $(2x - 7)(x + 5) = 0 \Rightarrow x = \dots$ $x = -5, 0, \frac{7}{2}$	M1 dM1 A1 (3)
(b)	$2(y-5)^6 + 3(y-5)^4 - 35(y-5)^2 = 0$ States that $y = 5$ is a solution $(y-5)^2 = \frac{7}{2} \Rightarrow y = \dots$ $y = 5 + \sqrt{\frac{7}{2}} \text{ or } y = 5 - \sqrt{\frac{7}{2}} \text{ or exact equivalent}$ $\text{Both } y = 5 + \sqrt{\frac{7}{2}} \text{ and } y = 5 - \sqrt{\frac{7}{2}} \text{ or exact equivalent.}$	B1 M1 A1ft A1 (4) (7 marks)

4 - (WMA11/P1(IAL)_Summer_2019_Q6) - Algebra And Functions

(a)	Sets $4x + c = x(x - 3)$ and attempts to write as a 3TQ Uses $b^2 = 4ac$ for their $x^2 - 7x - c = 0$ Correct equation $49 = -4c$ or $49 + 4c = 0$ $c = -12.25$ oe	M1 dM1 A1 A1 (4)
(b)	Attempt to solve $x^2 - 7x - c = 0$ with their c Attempt to find the y coordinate for their x coordinate $\left(\frac{7}{2}, \frac{7}{4}\right)$ oe	M1 dM1 A1 (3) (7 marks)

5 - (WMA11/P1(IAL)_Winter_2019_Q4) - Algebra And Functions

	<p>When ---- represents $<$ or $>$ and ——— represents \leq or \geq</p> <p>Either $2y \leq x$ or $y \geq 2x - \frac{1}{8}x^2$</p> <p>$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \dots$ or $x \leq \dots$</p> <p>$x < 16, 2y \leq x$ and $y \geq 2x - \frac{1}{8}x^2$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>(3 marks)</p>
Alt1	<p>When ---- represents \leq or \geq and ——— represents $<$ or $>$</p> <p>Either $2y < x$ or $y > 2x - \frac{1}{8}x^2$</p> <p>$2x - \frac{1}{8}x^2 = 0 \Rightarrow x = 16 \Rightarrow x < \dots$ or $x \leq \dots$</p> <p>$x \leq 16, 2y < x$ and $y > 2x - \frac{1}{8}x^2$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p>

6 - (WMA11/P1(IAL)_Winter_2019_Q8) - Algebra And Functions

(a)	States $y = 4$	B1	(1)
(b)	States (16, 9) only	B1	(1)
(c)	$k \leq 4, k = 9$	B1, B1	(2)
(d) (i)	$a = 6$	B1	(2)
(ii)	$y = f(x - 3)$	B1	(2)
		(6 marks)	