IB Diploma

MATHEMATICS

HL

Paper 2

2012 — 2020

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1 - (MAT-S 2012-Paper 2/1-Q6) - *FUNCTIONS - ROOTS*

[Maximum mark: 5]

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

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2 - (MAT-S 2014-Paper 2/2-Q7) - *FUNCTIONS - ROOTS, GRAPHS*

The function f is defined as $f(x) = -3 + \frac{1}{x-2}$, $x \neq 2$.

(a) (i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.

(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts. *[4]*

(b) Find the inverse function f^{-1}, stating its domain. *[4]*

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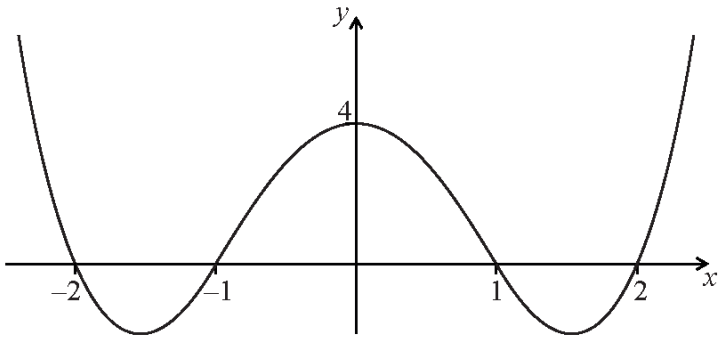
3 - (MAT-S 2014-Paper 2/1-Q12) - *FUNCTIONS - ROOTS, GRAPHS*

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.



(i) Find the value of $(f \circ g)(1)$.

(ii) Find the value of $(f \circ g \circ g)(1)$.

(iii) Sketch the graph of $y = (f \circ g)(x)$. *[5]*

(b) (i) Sketch the graph of $y = f(x)$.

(ii) State the zeros of f. *[3]*

(c) (i) Sketch the graph of $y = (f \circ f)(x)$.

(ii) State the zeros of $f \circ f$. *[3]*

(d) Given that we can denote $\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}}$ as f^n,

(i) find the zeros of f^3;

(ii) find the zeros of f^4;

(iii) deduce the zeros of f^8. *[3]*

(e) The zeros of f^{2n} are $a_1, a_2, a_3, \ldots, a_N$.

(i) State the relation between n and N;

(ii) Find, and simplify, an expression for $\sum_{r=1}^{N} |a_r|$ in terms of n. *[4]*

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4 - (MAT-S 2015-Paper 2/2-Q3) - *FUNCTIONS - ROOTS, GRAPHS*

(a) Sketch the graph of $y = (x-5)^2 - 2|x-5| - 9$, for $0 \le x \le 10$. [3]

(b) Hence, or otherwise, solve the equation $(x-5)^2 - 2|x-5| - 9 = 0$. [2]

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5 - (MAT-S 2015-Paper 2/1-Q10) - *FUNCTIONS - ROOTS*

A function f is defined by $f(x) = (x+1)(x-1)(x-5)$, $x \in \mathbb{R}$.

(a) Find the values of x for which $f(x) < |f(x)|$. [3]

A function g is defined by $g(x) = x^2 + x - 6$, $x \in \mathbb{R}$.

(b) Find the values of x for which $g(x) < \frac{1}{g(x)}$. [7]

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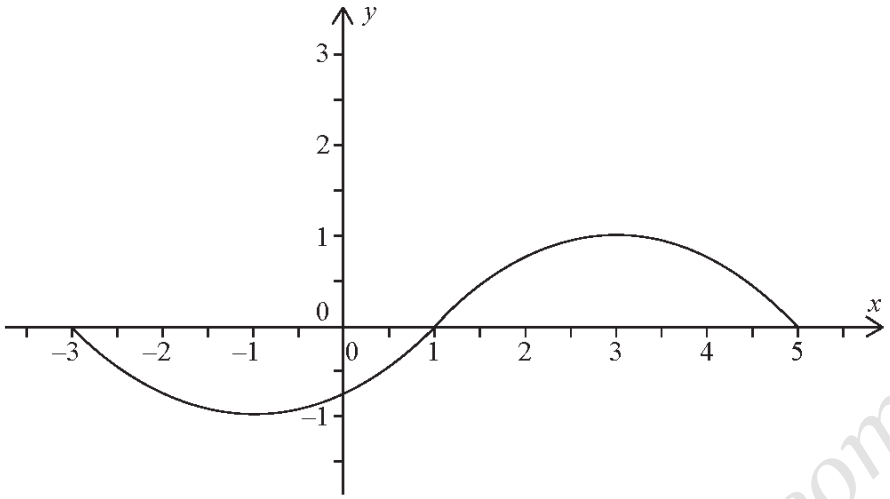
6 - (MAT-W 2015-Paper 2/0-Q12) - *FUNCTIONS - ROOTS*

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.
The function has a maximum at $(3, 1)$ and a minimum at $(-1, -1)$.



(a) The functions u and v are defined as $u(x) = x - 3$, $v(x) = 2x$ where $x \in \mathbb{R}$.

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$. [7]

(b) (i) Explain why f does not have an inverse.

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}.
State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints. [6]

Consider the function defined by $h(x) = \dfrac{2x-5}{x+d}$, $x \neq -d$ and $d \in \mathbb{R}$.

(c) (i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \dfrac{2x}{x+1}$, $x \neq -1$.

(iii) Find $k(x)$. [8]

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ANSWERS

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1 - (MAT-S 2012-Paper 2/1-Q6) - *FUNCTIONS - ROOTS*

$h(x) = f(x-3) - 2 = \ln(x-3) - 2$

$g(x) = -h(x) = 2 - \ln(x-3)$

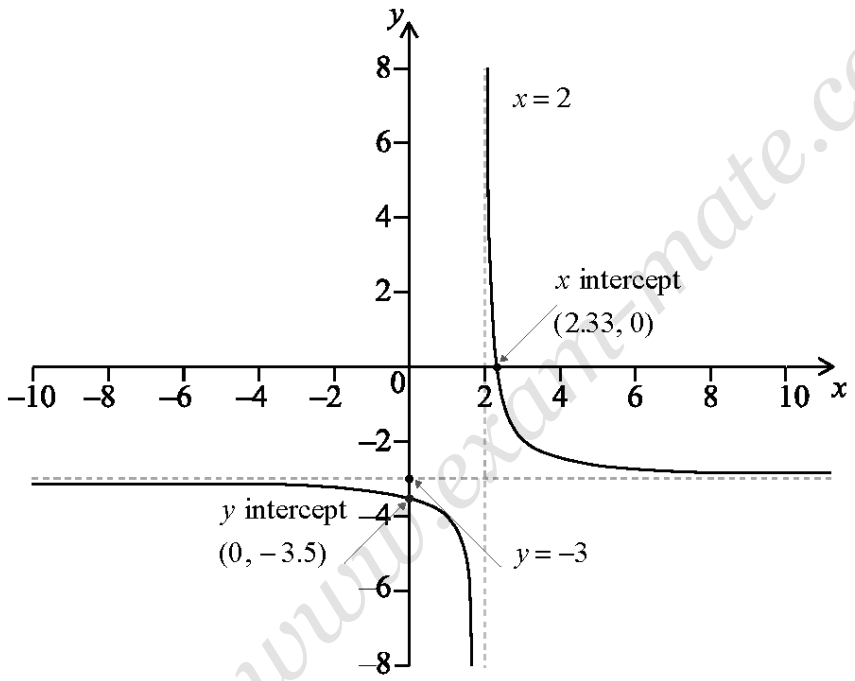
Note:	Award **M1** only if it is clear the effect of the reflection in the x-axis: the expression is correct ***OR*** there is a change of signs of the previous expression ***OR*** there's a graph or an explanation making it explicit

$= \ln e^2 - \ln(x-3)$

$= \ln\left(\frac{e^2}{x-3}\right)$

2 - (MAT-S 2014-Paper 2/2-Q7) - *FUNCTIONS - ROOTS, GRAPHS*

(a)



A1A1A1

Note:	Award ***A1*** for correct shape, ***A1*** for $x = 2$ clearly stated and ***A1*** for $y = -3$ clearly stated.

x intercept $(2.33, 0)$ and y intercept $(0, -3.5)$ ***A1***

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(b) $x = -3 + \frac{1}{y-2}$

Note: Award ***M1*** for interchanging x and y (can be done at a later stage).

$$x + 3 = \frac{1}{y-2}$$

$$y - 2 = \frac{1}{x+3}$$

Note: Award ***M1*** for attempting to make y the subject.

$$f^{-1}(x) = 2 + \frac{1}{x+3} \left(= \frac{2x+7}{x+3} \right), \; x \neq -3$$

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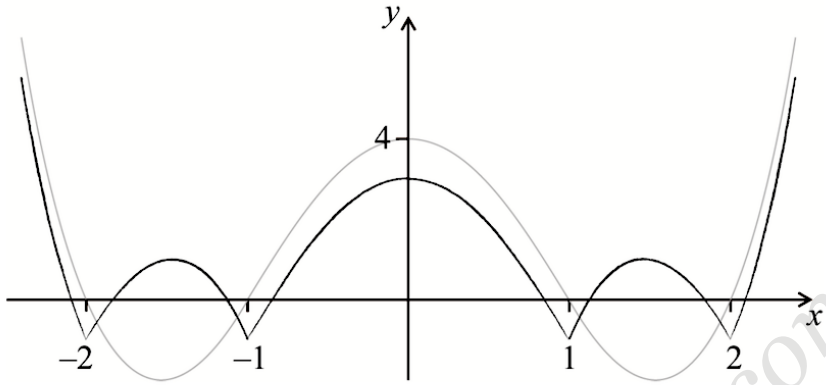
3 - (MAT-S 2014-Paper 2/1-Q12) - *FUNCTIONS - ROOTS, GRAPHS*

(a) (i) $f(0) = -1$

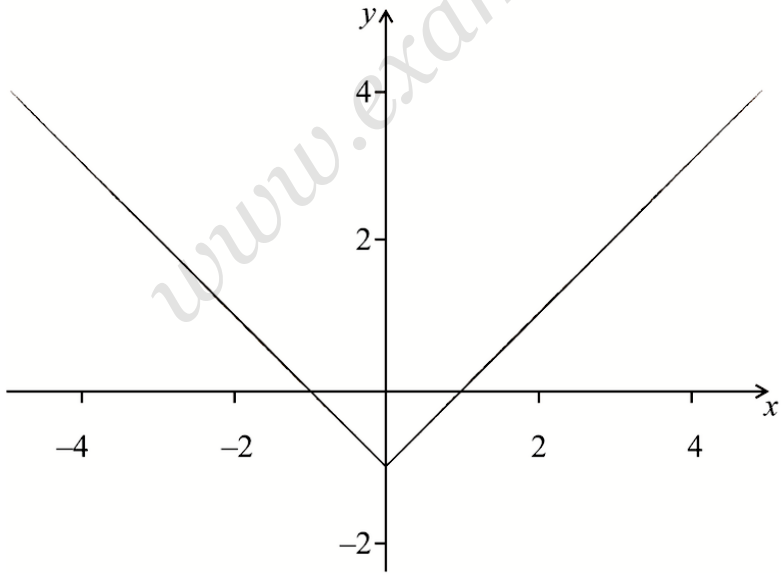
(ii) $(f \circ g)(0) = f(4) = 3$

(iii)



Note: Award ***M1*** for evidence that the lower part of the graph has been reflected and ***A1*** correct shape with y-intercept below 4.

(b) (i)



Note: Award ***M1*** for any translation of $y = |x|$.

(ii) ± 1

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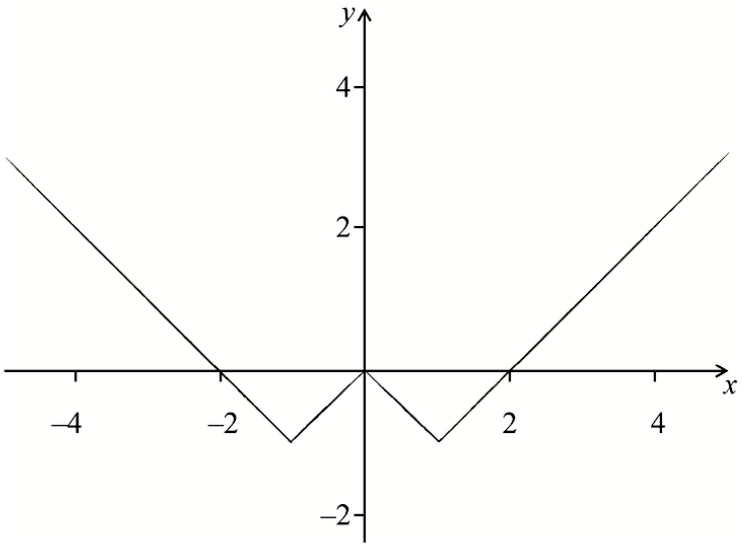
(c) (i)



Note:	Award ***M1*** for evidence that lower part of (b) has been reflected in the x-axis and translated.

(ii) $0,\ \pm 2$

(d) (i) $\pm 1,\ \pm 3$

(ii) $0,\ \pm 2,\ \pm 4$

(iii) $0,\ \pm 2,\ \pm 4,\ \pm 6,\ \pm 8$

(e) (i) $(1, 3), (2, 5), \ldots$

$N = 2n+1$

(ii) Using the formula of the sum of an arithmetic series

EITHER

$$4(1+2+3+\ldots+n) = \frac{4}{2}n(n+1)$$

$$= 2n(n+1)$$

OR

$$2(2+4+6+\ldots+2n) = \frac{2}{2}n(2n+2)$$

$$= 2n(n+1)$$

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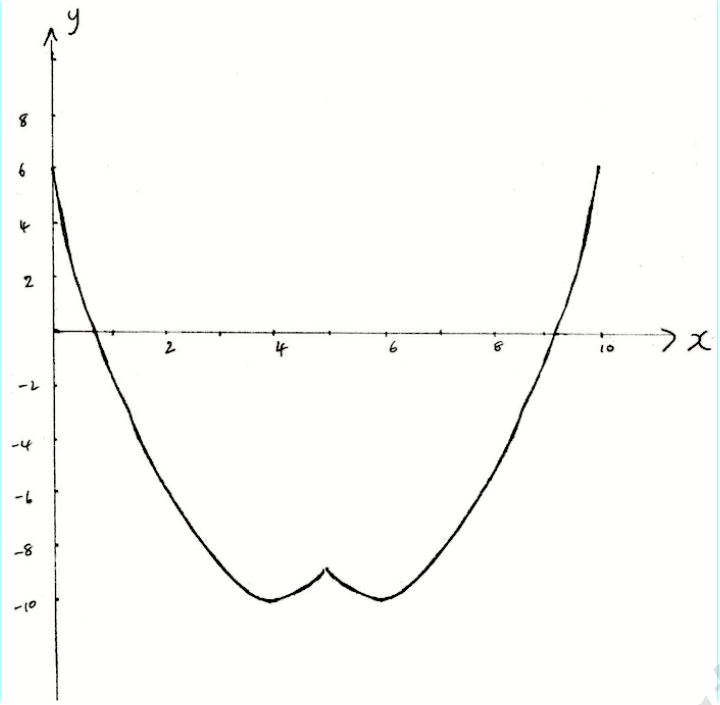
4 - (MAT-S 2015-Paper 2/2-Q3) - *FUNCTIONS - ROOTS, GRAPHS*

(a)



general shape including 2 minimums, cusp
correct domain and symmetrical about the middle $(x = 5)$

(b) $x = 9.16$ or $x = 0.838$

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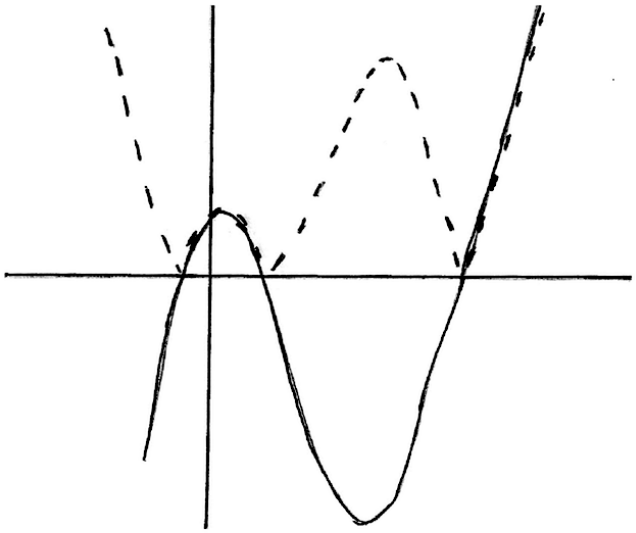
5 - (MAT-S 2015-Paper 2/1-Q10) - *FUNCTIONS - ROOTS*

(a)



as roots of $f(x)=0$ are $-1, 1, 5$

solution is $]-\infty,-1[\cup]1,5[$ $(x<-1 \text{ or } 1<x<5)$

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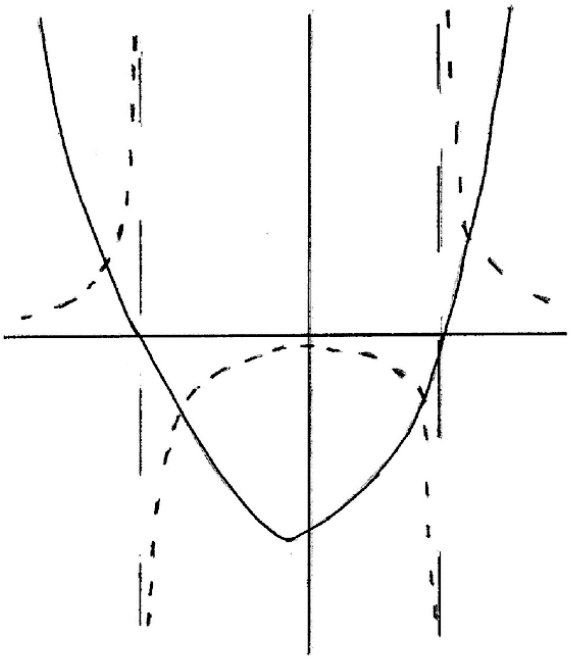
(b) METHOD 1

(graphs of $g(x)$ and $\frac{1}{g(x)}$)



roots of $g(x)=0$ are -3 and 2

Notes: Award ***M1*** if quadratic graph is drawn or two roots obtained.
Roots may be indicated anywhere *eg* asymptotes on graph or in inequalities below.

the intersections of the graphs $g(x)$ and of $1/g(x)$
are $-3.19, -2.79, 1.79, 2.19$

Note: Award ***A1*** for at least one of the values above seen anywhere.

solution is $]-3.19,-3[\cup]-2.79,1.79[\cup]2,2.19[$
$(-3.19 < x < -3$ or $-2.79 < x < 1.79$ or $2 < x < 2.19)$

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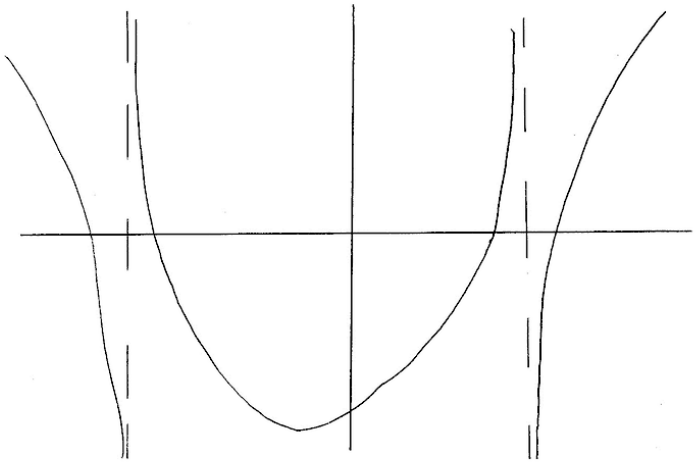
METHOD 2

(graph of $g(x) - \frac{1}{g(x)}$)



asymptotes at $x = -3$ and $x = 2$

Note: May be indicated on the graph.

roots of graph are $-3.19, -2.79, 1.79, 2.19$

Note: Award ***A1*** for at least one of the values above seen anywhere.

solution is (when graph is negative)
$]-3.19,-3[\cup]-2.79,1.79[\cup]2,2.19[$
$(-3.19 < x < -3$ or $-2.79 < x < 1.79$ or $2 < x < 2.19)$

6 - (MAT-W 2015-Paper 2/0-Q12) - *FUNCTIONS - ROOTS*

(a) (i) $-4 \le y \le -2$

(ii) $-5 \le y \le -1$

(iii) $-3 \le 2x - 6 \le 5$

Note: Award ***M1*** for $f(2x-6)$.

$3 \le 2x \le 11$

$\frac{3}{2} \le x \le \frac{11}{2}$

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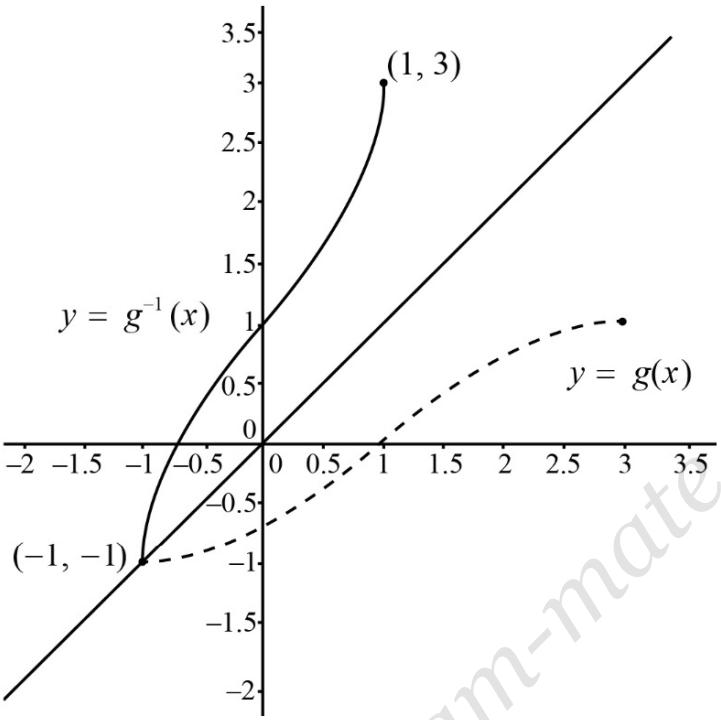
(b) (i) any valid argument *eg* f is not one to one, f is many to one, fails horizontal line test, not injective ***R1***

(ii) largest domain for the function $g(x)$ to have an inverse is $[-1, 3]$ ***A1A1***

(iii)



y-intercept indicated (coordinates not required) ***A1***
correct shape ***A1***
coordinates of end points $(1, 3)$ and $(-1, -1)$ ***A1***

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(c) (i) $y = \dfrac{2x-5}{x+d}$

$(x+d)\,y = 2x-5$

Note: Award ***M1*** for attempting to rearrange x and y in a linear expression.

$x(y-2) = -dy-5$

$x = \dfrac{-dy-5}{y-2}$

Note: x and y can be interchanged at any stage

$h^{-1}(x) = \dfrac{-dx-5}{x-2}$

Note: Award ***A1*** only if $h^{-1}(x)$ is seen.

(ii) self Inverse $\Rightarrow h(x) = h^{-1}(x)$

$\dfrac{2x-5}{x+d} \equiv \dfrac{-dx-5}{x-2}$

$d = -2$

(iii) **METHOD 1**

$\dfrac{2k(x)-5}{k(x)-2} = \dfrac{2x}{x+1}$

$k(x) = \dfrac{x+5}{2}$

METHOD 2

$h^{-1}\left(\dfrac{2x}{x+1}\right) = \dfrac{2\left(\dfrac{2x}{x+1}\right)-5}{\dfrac{2x}{x+1}-2}$

$k(x) = \dfrac{x+5}{2}$

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