Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 1 2 1 4 6 4 7 7 5 3 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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[Turn over

1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

$2 \cdot \frac{3^1}{3^{2x}} < 5^x \rightarrow \frac{6}{9^x} < 5^x \rightarrow 6 < 9^x \cdot 5^x \rightarrow 6 < 45^x$

$\rightarrow \log_{45} 6 < x \rightarrow$ OR $\rightarrow x > \frac{\ln 45}{\ln 6}$ #

2 **(a)** Expand $(2-3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

$$(2-3x)^{-2} = \left[2\left(1-\frac{3}{2}x\right)\right]^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2} = \frac{1}{4}\left(1-\frac{3}{2}x\right)^{-2}$$

$$= \frac{1}{4}\left(1+(-2)\left(-\frac{3}{2}x\right) + \frac{(-2)(-3)}{2}\left(-\frac{3}{2}x\right)^2 + \cdots\right)$$

$$= \frac{1}{4}\left(1+3x+\frac{27}{4}x^2+\cdots\right) = \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\cdots \quad ※$$

(b) State the set of values of x for which the expansion is valid. [1]

$$-1 < \frac{3}{2}x < 1 \Rightarrow -2 < 3x < 2 \Rightarrow -\frac{2}{3} < x < \frac{2}{3} \quad ※$$

3 Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

$$\frac{\tan\theta + \tan 60°}{1 - \tan\theta . \tan 60°} = 2 + \frac{\tan 60° - \tan\theta}{1 + \tan\theta . \tan 60°}$$

$$\Rightarrow \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} = 2 + \frac{\sqrt{3} - \tan\theta}{1 + \sqrt{3}\tan\theta}$$

$$\Rightarrow \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} = \frac{2 + 2\sqrt{3}\tan\theta + \sqrt{3} - \tan\theta}{1 + \sqrt{3}\tan\theta}$$

$$\Rightarrow (1 - \sqrt{3}\tan\theta)(2 + 2\sqrt{3}\tan\theta + \sqrt{3} - \tan\theta) = (1 + \sqrt{3}\tan\theta)(\tan\theta + \sqrt{3})$$

$$\Rightarrow 2 + \cancel{2\sqrt{3}\tan\theta} + \sqrt{3} - \tan\theta - \cancel{2\sqrt{3}\tan\theta} - 6\tan^2\theta - 3\tan\theta + \cancel{\sqrt{3}\tan^2\theta} =$$
$$\tan\theta + \sqrt{3} + \cancel{\sqrt{3}\tan^2\theta} + 3\tan\theta$$

$$\Rightarrow 3\tan^2\theta + 4\tan\theta - 1 = 0 \quad \Rightarrow \tan\theta = \frac{-4 \pm \sqrt{16 + 12}}{6}$$

$$\Rightarrow \theta_1 = 12.1° \qquad \theta_2 = -57°.14 + 180° = 122.9°$$