A LEVEL Cambridge Topical Past Papers

FURTHER MATHEMATICS 2

Further Statistics

2012 — 2019

1 - (9231-S 2012-Paper 2/2-Q6) **-** *Further work on distributions*

The probability that a particular type of light bulb is defective is 0.01. A large number of these bulbs are tested, one by one. Assuming independence, find the probability that the tenth bulb tested is the first to be found defective. [2]

The first defective bulb is the Nth to be tested. Write down the value of $\mathrm{E}(N)$. [1]

Find the least value of n such that $\mathrm{P}(N \leqslant n)$ is greater than 0.9. [3]

2 - (9231-S 2012-Paper 2/3-Q7) **-** *Further work on distributions*

The waiting time, T minutes, before a customer is served in a restaurant has distribution function F given by

$$F(t) = \begin{cases} 1 - e^{-\lambda t} & t \geqslant 0, \\ 0 & t < 0, \end{cases}$$

where λ is a positive constant. The standard deviation of T is 8. Find

(i) the value of λ, [2]

(ii) the probability that a customer has to wait between 5 and 10 minutes before being served, [2]

(iii) the median value of T. [3]

3 - (9231-S 2012-Paper 2/2-Q8) **-** *Further work on distributions*

The number of flaws in a randomly chosen 100 metre length of ribbon is modelled by a Poisson distribution with mean 1.6. The random variable X metres is the distance between two successive flaws. Show that the distribution function of X is given by

$$\mathrm{F}(x) = \begin{cases} 1 - \mathrm{e}^{-0.016x} & x \geqslant 0, \\ 0 & x < 0, \end{cases}$$

and deduce that X has a negative exponential distribution, stating its mean. [4]

Find

(i) the median distance between two successive flaws, [3]

(ii) the probability that there is a distance of at least 50 metres between two successive flaws. [2]

4 - (9231-S 2012-Paper 2/3-Q9) **-** *Further work on distributions*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{2a} & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. Find the distribution function of X. [3]

The random variable Y is defined by $Y = e^X$. Find the distribution function of Y. [3]

Given that $a = 4$, find the value of k for which $P(Y \geqslant k) = 0.25$. [3]

5 - (9231-W 2012-Paper 2/1-Q6) **-** *Further work on distributions*

In a skiing resort, for each day during the winter season, the probability that snow will fall on that day is 0.2, independently of any other day. The first day of the winter season is 1 December. Find, for the winter season,

(i) the probability that the first snow falls on 20 December, [2]

(ii) the probability that the first snow falls before 5 December, [2]

(iii) the earliest date in December such that the probability that the first snow falls on or before that date is at least 0.95. [3]

6 - (9231-W 2012-Paper 2/3-Q6) **-** *Further work on distributions*

The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}\mathrm{e}^{-\frac{1}{6}x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the distribution function of X, [2]

(ii) the probability that X lies between the median and the mean. [4]

7 - (9231-W 2012-Paper 2/1-Q7) **-** *Further work on distributions*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{2}{15}x & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^3$. Show that the distribution function G of Y is given by

$$G(y) = \begin{cases} 0 & y < 1, \\ \frac{1}{15}\left(y^{\frac{2}{3}} - 1\right) & 1 \leqslant y \leqslant 64, \\ 1 & y > 64. \end{cases}$$ [4]

Find

(i) the median value of Y, [3]

(ii) $E(Y)$. [4]

8 - (9231-W 2012-Paper 2/3-Q11) **-** *Simple harmonic motion, Further work on distributions*

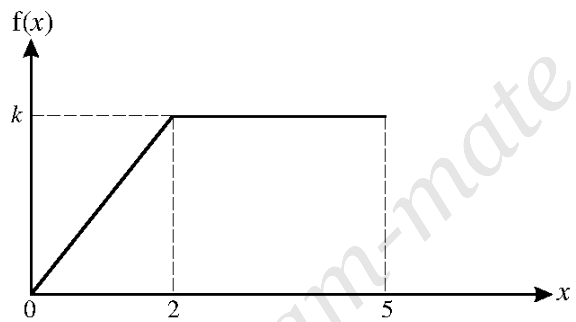
Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light elastic string of modulus of elasticity $8mg$ and natural length a. The other end of the string is attached to a fixed point O. The particle is pulled vertically downwards a distance $\frac{1}{4}a$ from its equilibrium position and released from rest. Show that the string first becomes slack after a time $\frac{2\pi}{3}\sqrt{\left(\frac{a}{8g}\right)}$. [8]

Find, in terms of a, the total distance travelled by P from its release until it subsequently comes to instantaneous rest for the first time. [6]

OR



The continuous random variable X takes values in the interval $0 \leqslant x \leqslant 5$ only. For $0 \leqslant x \leqslant 5$ the graph of its probability density function f consists of two straight line segments, as shown in the diagram. Find k and show that f is given by

$$f(x) = \begin{cases} \frac{1}{8}x & 0 \leqslant x \leqslant 2, \\ \frac{1}{4} & 2 < x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

The random variable Y is given by $Y = X^2$.

(i) Find the probability density function of Y. [6]

(ii) Show that $E(Y) = 10.25$. [3]

(iii) Show that the median of Y is the square of the median of X. [2]

9 - (9231-S 2013-Paper 2/1-Q6) **-** *Further work on distributions*

The random variable X has distribution function F given by

$$F(x) = \begin{cases} 1 - e^{-0.6x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Identify the distribution of X and state its mean. [2]

Find

(i) $P(X > 4)$, [2]

(ii) the median of X. [3]

10 - (9231-S 2013-Paper 2/1-Q8) **-** *Further work on distributions*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}x & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^3$. Show that Y has probability density function g given by

$$g(y) = \begin{cases} \frac{1}{18}y^{-\frac{1}{3}} & 8 \leqslant y \leqslant 64, \\ 0 & \text{otherwise.} \end{cases}$$ [6]

Find E(Y). [3]