A LEVEL Cambridge Topical Past Papers

FURTHER MATHEMATICS 2

Further Mechanics

2012 — 2019

1 - (9231-S 2012-Paper 2/3-Q1) - *Momentum and impulse*

Two smooth spheres A and B, of equal radii and of masses $3m$ and $6m$ respectively, are at rest on a smooth horizontal surface. Sphere A is projected directly towards B with speed u. The coefficient of restitution between A and B is e. Show that the kinetic energy lost in the collision between A and B is $mu^2(1 - e^2)$. [7]

2 - (9231-S 2012-Paper 2/2-Q2) - *Momentum and impulse*

Two particles, of masses $3m$ and m, are moving in the same straight line towards each other with speeds $2u$ and u respectively. When they collide, the impulse acting on each particle has magnitude $4mu$. Show that the total loss in kinetic energy is $\frac{4}{3}mu^2$. [6]

3 - (9231-S 2012-Paper 2/2-Q11) - *Momentum and impulse, Simple harmonic motion, Bivariate data*

Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light elastic string of modulus of elasticity $4mg$ and natural length l. The other end of the string is attached to a fixed point O. The particle rests in equilibrium at the point E, vertically below O. The particle is pulled down a vertical distance $\frac{1}{8}l$ from E and released from rest. Show that the motion of P is simple harmonic with period $\pi\sqrt{\left(\frac{l}{g}\right)}$. [4]

At an instant when P is moving vertically downwards through E, the string is cut. When P has descended a further distance $\frac{7}{16}l$ under gravity, it strikes a fixed smooth plane which is inclined at $30°$ to the horizontal. The coefficient of restitution between P and the plane is $\frac{1}{3}$. Show that the speed of P immediately after the impact is $\frac{1}{4}\sqrt{(5gl)}$. [8]

OR

A new restaurant S has recently opened in a particular town. In order to investigate any effect of S on an existing restaurant R, the daily takings, x and y in thousands of dollars, at R and S respectively are recorded for a random sample of 8 days during a six-month period. The results are shown in the following table.

Day	1	2	3	4	5	6	7	8
x	1.2	1.4	0.9	1.1	0.8	1.0	0.6	1.5
y	0.3	0.4	0.6	0.6	0.25	0.75	0.6	0.35

(i) Calculate the product moment correlation coefficient for this sample. [4]

(ii) Stating your hypotheses, test, at the 2.5% significance level, whether there is negative correlation between daily takings at the two restaurants and comment on your result in the context of the question. [5]

Another sample is taken over N randomly chosen days and the product moment correlation coefficient is found to be -0.431. A test, at the 5% significance level, shows that there is evidence of negative correlation between daily takings in the two restaurants.

(iii) Find the range of possible values of N. [3]

4 - (9231-W 2012-Paper 2/1-Q4) - *Momentum and impulse*

A particle P of mass $2m$, moving on a smooth horizontal plane with speed u, strikes a fixed smooth vertical barrier. Immediately before the collision the angle between the direction of motion of P and the barrier is 60°. The coefficient of restitution between P and the barrier is $\frac{1}{3}$. Show that P loses two-thirds of its kinetic energy in the collision. [5]

Subsequently P collides directly with a particle Q of mass m which is moving on the plane with speed u towards P. The magnitude of the impulse acting on each particle in the collision is $\frac{2}{3}mu(1+\sqrt{3})$.

(i) Show that the speed of P after this collision is $\frac{1}{3}u$. [2]

(ii) Find the exact value of the coefficient of restitution between P and Q. [4]

5 - (9231-W 2012-Paper 2/3-Q4) - *Momentum and impulse*

Three particles A, B and C have masses m, $2m$ and m respectively. The particles are able to move on a smooth horizontal surface in a straight line, and B is between A and C. Initially A is moving towards B with speed $2u$ and C is moving towards B with speed u. The particle B is at rest. The coefficient of restitution between any pair of particles is e. The first collision is between A and B.

(i) Show that the speed of B immediately before its collision with C is $\frac{2}{3}u(1+e)$. [4]

(ii) Find the velocity of B immediately after its collision with C. [3]

(iii) Given that $e > \frac{1}{2}$, show that there are no further collisions between the particles. [4]

6 - (9231-S 2013-Paper 2/3-Q1) - *Momentum and impulse*

A bullet of mass m kg is fired into a fixed vertical barrier. It enters the barrier horizontally with speed $280\,\text{m}\,\text{s}^{-1}$ and emerges horizontally after $0.01\,\text{s}$ with speed $30\,\text{m}\,\text{s}^{-1}$. There is a constant horizontal resisting force of magnitude $1500\,\text{N}$. Find m. [4]

7 - (9231-S 2013-Paper 2/1-Q2) - *Momentum and impulse*

Three uniform small smooth spheres, A, B and C, have equal radii. Their masses are $4m$, $2m$ and m respectively. They lie in a straight line on a smooth horizontal surface with B between A and C. Initially A is moving towards B with speed u, B is at rest and C is moving in the same direction as A with speed $\frac{1}{2}u$. The coefficient of restitution between any two of the spheres is e. The first collision is between A and B. In this collision sphere A loses three-quarters of its kinetic energy. Show that $e = \frac{1}{2}$. [6]

Find the speed of B after its collision with C and deduce that there are no further collisions between the spheres. [5]

8 - (9231-S 2013-Paper 2/3-Q3) - *Momentum and impulse*

Two uniform small smooth spheres A and B, of masses m and $2m$ respectively, and with equal radii, are at rest on a smooth horizontal surface. Sphere A is projected directly towards B with speed u, and collides with B. After this collision, sphere B collides directly with a fixed smooth vertical barrier. The total kinetic energy of the spheres after this second collision is equal to one-ninth of its value before the first collision. Given that the coefficient of restitution between B and the barrier is 0.5, find the coefficient of restitution between A and B. [9]

9 - (9231-W 2013-Paper 2/3-Q2) - *Momentum and impulse*

Three uniform small smooth spheres A, B and C, of equal radii and of masses $4m$, λm and m respectively, are at rest in a straight line on a smooth horizontal plane, with B between A and C. Sphere A is projected directly towards B with speed u. The coefficient of restitution between A and B, and between B and C, is $\frac{1}{2}$. Show that the speed of B after it is struck by A is $\frac{6u}{\lambda + 4}$. [4]

Given that the speed of C after it is struck by B is u, find the value of λ. [5]

10 - (9231-W 2013-Paper 2/1-Q5) - *Momentum and impulse*

Two uniform small smooth spheres A and B, of equal radii, have masses $2m$ and m respectively. They lie at rest on a smooth horizontal plane. Sphere A is projected directly towards B with speed u. After the collision B goes on to collide directly with a fixed smooth vertical barrier, before colliding with A again. The coefficient of restitution between A and B is $\frac{2}{3}$ and the coefficient of restitution between B and the barrier is e. After the second collision between A and B, the speed of B is five times the speed of A. Find the two possible values of e. [11]