1 **(i)** Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^5$. [2]

$$(1-2x)^5 = {}^5C_0(1)^5(-2x)^0 + {}^5C_1(1)^4(-2x)^1 + {}^5C_2(1)^3(-2x)^2 + \cdots$$

$$= 1 - 10x + 40x^2 + \cdots$$

(ii) Given that the coefficient of x^2 in the expansion of $(1+ax+2x^2)(1-2x)^5$ is 12, find the value of the constant a. [3]

$$(1+ax+2x^2)(1-2x)^5 = (1+ax+2x^2)(1-10x+40x^2+\cdots)$$

$$= 40x^2 - 10ax^2 + 2x^2 + \cdots$$

$$= (42-10a)x^2 + \cdots$$

$$42 - 10a = 12 \rightarrow 30 = 10a \rightarrow a = 3 \text{ ※}$$

2 A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

$$y = 2x + 5x^{-1} \rightarrow dy/dx = 2 - 5x^{-2} = 2 - \frac{5}{x^2} \qquad dx/dt = 0.02 \text{ units/sec}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow \frac{dy}{dt} = \left(2 - \frac{5}{x^2}\right) \times 0.02$$

$$x = 1 \rightarrow \frac{dy}{dt} = \left(2 - \frac{5}{1^2}\right) \times 0.02 \rightarrow \frac{dy}{dt} = -0.06 \text{ units/sec}$$

decreasing ※

3 A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point $(1, 1)$ lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

$\frac{dy}{dx} = 12(2x+1)^{-2} \rightarrow y = \int 12(2x+1)^{-2}\,dx \rightarrow y = \frac{12}{2(-1)}(2x+1)^{-1} + C$

$y = \frac{-6}{2x+1} + C$ (1,1) lies on the curve.

$1 = \frac{-6}{2+1} + C \rightarrow C = 3 \quad \rightarrow y = \frac{-6}{2x+1} + 3$

x-int: $y = 0 \rightarrow \frac{-6}{2x+1} = -3 \rightarrow -6 = -3(2x+1) \rightarrow 2x+1 = \frac{-6}{-3}$

$\rightarrow 2x+1 = 2 \rightarrow x = 1/2 \quad (\frac{1}{2}, 0)$ #

4 **(i)** Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta)$ expand

$= \sin\theta - \sin^2\theta\cos\theta + \cos\theta - \sin\theta\cos^2\theta$

$= \sin\theta - (1-\cos^2\theta)\cos\theta + \cos\theta - \sin\theta(1-\sin^2\theta)$

$= \cancel{\sin\theta} - \cancel{\cos\theta} + \cos^3\theta + \cancel{\cos\theta} - \cancel{\sin\theta} + \sin^3\theta$

$= \sin^3\theta + \cos^3\theta$ #

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

$\sin^3\theta + \cos^3\theta = 3\cos^3\theta \rightarrow \sin^3\theta = 2\cos^3\theta \rightarrow \frac{\sin^3\theta}{\cos^3\theta} = 2$

$\rightarrow \tan^3\theta = 2 \rightarrow \tan\theta = \sqrt[3]{2} \rightarrow \theta_1 = 51.6°$ $\quad \theta_2 = 180 + 51.6° = 231.6°$