A LEVEL Cambridge Topical Past Papers

FURTHER MATHEMATICS 1

2012 — 2019

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1 - (9231-S 2012-Paper 1/1-Q1) **-** *Roots of polynomial equations*

The roots of the cubic equation $x^3 - 7x^2 + 2x - 3 = 0$ are α, β, γ. Find the values of

(i) $\alpha^2 + \beta^2 + \gamma^2$, [2]

(ii) $\alpha^3 + \beta^3 + \gamma^3$. [3]

2 - (9231-S 2012-Paper 1/3-Q8) - *Roots of polynomial equations*

The cubic equation $x^3 - x^2 - 3x - 10 = 0$ has roots α, β, γ.

(i) Let $u = -\alpha + \beta + \gamma$. Show that $u + 2\alpha = 1$, and hence find a cubic equation having roots $-\alpha + \beta + \gamma$, $\alpha - \beta + \gamma$, $\alpha + \beta - \gamma$. [5]

(ii) State the value of $\alpha\beta\gamma$ and hence find a cubic equation having roots $\frac{1}{\beta\gamma}$, $\frac{1}{\gamma\alpha}$, $\frac{1}{\alpha\beta}$. [5]

3 - (9231-W 2012-Paper 1/3-Q7) - *Roots of polynomial equations*

A cubic equation has roots α, β and γ such that

$$\alpha + \beta + \gamma = 4,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 14,$$
$$\alpha^3 + \beta^3 + \gamma^3 = 34.$$

Find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

Show that the cubic equation is

$$x^3 - 4x^2 + x + 6 = 0,$$

and solve this equation. [6]

4 - (9231-W 2012-Paper 1/1-Q11) - *Roots of polynomial equations, Matrices*

Answer only **one** of the following two alternatives.

EITHER

The roots of the equation $x^4 - 3x^2 + 5x - 2 = 0$ are $\alpha, \beta, \gamma, \delta$, and $\alpha^n + \beta^n + \gamma^n + \delta^n$ is denoted by S_n. Show that

$$S_{n+4} - 3S_{n+2} + 5S_{n+1} - 2S_n = 0.$$ [2]

Find the values of

(i) S_2 and S_4, [3]

(ii) S_3 and S_5. [6]

Hence find the value of

$$\alpha^2(\beta^3 + \gamma^3 + \delta^3) + \beta^2(\gamma^3 + \delta^3 + \alpha^3) + \gamma^2(\delta^3 + \alpha^3 + \beta^3) + \delta^2(\alpha^3 + \beta^3 + \gamma^3).$$ [3]

OR

The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -1 & 4 \\ 3 & 4 & 6 & 1 \\ -1 & 2 & 8 & -7 \end{pmatrix}.$$

The range space of T is R. In any order,

(i) show that the dimension of R is 2,

(ii) find a basis for R and obtain a cartesian equation for R,

(iii) find a basis for the null space of T.

[9]

The vector $\begin{pmatrix} 8 \\ 7 \\ k \end{pmatrix}$ belongs to R. Find the value of k and, with this value of k, find the general solution of

$$\mathbf{Mx} = \begin{pmatrix} 8 \\ 7 \\ k \end{pmatrix}.$$ [5]

5 - (9231-S 2013-Paper 1/3-Q2) - *Roots of polynomial equations*

The roots of the equation $x^4 - 4x^2 + 3x - 2 = 0$ are α, β, γ and δ; the sum $\alpha^n + \beta^n + \gamma^n + \delta^n$ is denoted by S_n. By using the relation $y = x^2$, or otherwise, show that α^2, β^2, γ^2 and δ^2 are the roots of the equation

$$y^4 - 8y^3 + 12y^2 + 7y + 4 = 0.$$ [3]

State the value of S_2 and hence show that

$$S_8 = 8S_6 - 12S_4 - 72.$$ [3]

6 - (9231-S 2013-Paper 1/1-Q3) **-** *Roots of polynomial equations*

The cubic equation $x^3 - 2x^2 - 3x + 4 = 0$ has roots α, β, γ. Given that $c = \alpha + \beta + \gamma$, state the value of c. [1]

Use the substitution $y = c - x$ to find a cubic equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$. [3]

Find a cubic equation whose roots are $\frac{1}{\alpha + \beta}$, $\frac{1}{\beta + \gamma}$, $\frac{1}{\gamma + \alpha}$. [2]

Hence evaluate $\frac{1}{(\alpha + \beta)^2} + \frac{1}{(\beta + \gamma)^2} + \frac{1}{(\gamma + \alpha)^2}$. [2]

7 - (9231-W 2013-Paper 1/1-Q2) - *Roots of polynomial equations*

The cubic equation $x^3 - px - q = 0$, where p and q are constants, has roots α, β, γ. Show that

(i) $\alpha^2 + \beta^2 + \gamma^2 = 2p$, [1]

(ii) $\alpha^3 + \beta^3 + \gamma^3 = 3q$, [2]

(iii) $6(\alpha^5 + \beta^5 + \gamma^5) = 5(\alpha^3 + \beta^3 + \gamma^3)(\alpha^2 + \beta^2 + \gamma^2)$. [3]

8 - (9231-W 2013-Paper 1/3-Q5) - *Roots of polynomial equations*

The equation

$$8x^3 + 36x^2 + kx - 21 = 0,$$

where k is a constant, has roots $a - d$, a, $a + d$. Find the numerical values of the roots and determine the value of k. [8]

9 - (9231-S 2014-Paper 1/1-Q1) **-** *Roots of polynomial equations*

The equation $x^3 + px + q = 0$, where p and q are constants, with $q \neq 0$, has one root which is the reciprocal of another root. Prove that $p + q^2 = 1$. [5]

10 - (9231-W 2014-Paper 1/1-Q11) - *Roots of polynomial equations, Matrices*

Answer only **one** of the following two alternatives.

EITHER

The roots of the quartic equation $x^4 + 4x^3 + 2x^2 - 4x + 1 = 0$ are α, β, γ and δ. Find the values of

(i) $\alpha + \beta + \gamma + \delta$, [1]

(ii) $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$, [2]

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$, [2]

(iv) $\dfrac{\alpha}{\beta\gamma\delta} + \dfrac{\beta}{\alpha\gamma\delta} + \dfrac{\gamma}{\alpha\beta\delta} + \dfrac{\delta}{\alpha\beta\gamma}$. [2]

Using the substitution $y = x + 1$, find a quartic equation in y. Solve this quartic equation and hence find the roots of the equation $x^4 + 4x^3 + 2x^2 - 4x + 1 = 0$. [7]

OR

The square matrix **A** has λ as an eigenvalue with **e** as a corresponding eigenvector. Show that if **A** is non-singular then

(i) $\lambda \neq 0$, [2]

(ii) the matrix $\mathbf{A}^{-1}$ has λ^{-1} as an eigenvalue with **e** as a corresponding eigenvector. [2]

The 3×3 matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} -2 & 2 & -4 \\ 0 & -1 & 5 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = (\mathbf{A} + 3\mathbf{I})^{-1},$$

where **I** is the 3×3 identity matrix. Find a non-singular matrix **P**, and a diagonal matrix **D**, such that $\mathbf{B} = \mathbf{PDP}^{-1}$. [10]

ANSWERS

1 - (9231-S 2012-Paper 1/1-Q1) **-** *Roots of polynomial equations*

States $\sum\alpha$ and $\sum\alpha\beta$ Uses formula for correctly.	$\sum\alpha = 7 \quad \sum\alpha\beta = 2$ $\sum\alpha^2 = 7^2 - 2\times 2 = 45$	B1 B1	2	
Uses formula for $\sum\alpha^3$ to obtain result.	$\sum\alpha^3 = 7\sum\alpha^2 - 2\sum\alpha + 9$ $= 315\text{-}14\text{+}9 = 310$	M1 A1A1	3	**[5]**

2 - (9231-S 2012-Paper 1/3-Q8) **-** *Roots of polynomial equations*

(i)	Deduces initial result.	$u = -\alpha + \beta + \gamma \Rightarrow u + 2\alpha = \alpha + \beta + \gamma = 1$	M1A1		
	Substitutes into cubic equation.	$\Rightarrow \alpha = \left(\dfrac{1-u}{2}\right)$	M1		
	Deduces new cubic equation.	$\Rightarrow \left(\dfrac{1-u}{2}\right)^3 - \left(\dfrac{1-u}{2}\right)^2 - 3\left(\dfrac{1-u}{2}\right) - 10 = 0$ $\Rightarrow \ldots \Rightarrow u^3 - u^2 - 13u + 93 = 0$	A1 A1	5	
(ii)	Deduces initial result.	$\alpha\beta\gamma = 10$ $\Rightarrow v = \dfrac{1}{\beta\gamma} \Rightarrow \dfrac{v}{\alpha} = \dfrac{1}{\alpha\beta\gamma} = \dfrac{1}{10} \Rightarrow \alpha = 10v$	B1 M1A1		
	Substitutes into cubic equation.	$(10v)^3 - (10v)^2 - 3(10v) - 10 = 0$	M1		
	Deduces new cubic equation.	$\Rightarrow 100v^3 - 10v^2 - 3v - 1 = 0$	A1	5	**[10]**
	Alternatively:				
(i)	**For final 3 marks in (i):** Award M1 for an attempt at formulae for all three coefficients. A1 for any two correct. A1 for completion	Let equation be $u^3 + bu^2 + cu + d = 0$. $-b = \sum\alpha = 1 \Rightarrow b = -1$ $c = 4\sum\alpha\beta - (\sum\alpha)^2$ $= 4\times(-3) - 1^2 = -13$ $-d = 4\sum\alpha\sum\alpha\beta - \left(\sum\alpha\right)^3 - 8\alpha\beta\gamma$ $= 4\times 1\times(-3) - 1^3 - 8\times 10 = -93$ So $u^3 - u^2 - 13u + 93 = 0$		(5)	
(ii)	**For final 4 marks in (ii):** Award M1 for an attempt at formulae for all three coefficients. A1 for any one correct. A1 for a second one correct. A1 for completion.	Let equation be $v^3 + bv^2 + cv + d = 0$. $-b = \dfrac{\Sigma\alpha}{\alpha\beta\gamma} = \dfrac{1}{10} \Rightarrow b = -\dfrac{1}{10}$ $c = \dfrac{\Sigma\alpha\beta}{(\alpha\beta\gamma)^2} = \dfrac{-3}{10^2} = -\dfrac{3}{100}$ $-d = \dfrac{1}{(\alpha\beta\gamma)^2} = \dfrac{1}{10^2} \Rightarrow d = -\dfrac{1}{100}$ So $v^3 - \dfrac{1}{10}v^2 - \dfrac{3}{100}v - \dfrac{1}{100} = 0$ or $100v^3 - 10v^2 - 3v - 1 = 0$.		(5)	**[10]**

3 - (9231-W 2012-Paper 1/3-Q7) **-** *Roots of polynomial equations*

$(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha) = \alpha^2+\beta^2+\gamma^2 \Rightarrow \sum\alpha\beta = 1$	M1A1	2	
Either			
Required equation is $x^3 - 4x^2 + x + c = 0$	M1		
$\Rightarrow \sum\alpha^3 - 4\sum\alpha^2 + 4 + 3c = 0$	M1		
$\Rightarrow 3c = 56 - 34 - 4 = 18 \Rightarrow c = 6$ (AG)	A1		
Or			
$\alpha^3+\beta^3+\gamma^3 - 3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$	(M1)		
(or some other appropriate identity, e.g.			
$(\alpha+\beta+\gamma)^3 = \alpha^3+\beta^3+\gamma^3+3(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)-3\alpha\beta\gamma$)	(M1A1)		
$\Rightarrow \ldots \Rightarrow \alpha\beta\gamma = -6$			
$\Rightarrow x^3 - 4x^2 + x + 6 = 0$ (AG)	A1		
$\Rightarrow (x+1)(x-2)(x-3) = 0 \Rightarrow x = -1, 2, 3.$	M1A1	6	**[8]**

4 - (9231-W 2012-Paper 1/1-Q11) **-** *Roots of polynomial equations, Matrices*

	EITHER				
	Substitute α into equation.	α is a root $\Rightarrow \alpha^4 - 3\alpha^2 + 5\alpha - 2 = 0$			
	Multiply by α^n. Obtain result.	$\Rightarrow \alpha^{n+4} - 3\alpha^{n+2} + 5\alpha^{n+1} - 2\alpha^n = 0$			
		Repeat for β, γ, δ and sum	M1		
		$\Rightarrow S_{n+4} - 3S_{n+2} + 5S_{n+1} - 2S_n = 0$ (AG)	A1	2	
(i)	Uses $\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$	$S_2 = 0 - 2\times(-3) = 6$	B1		
	Finds S_4 from formula.	$S_4 = 3\times6 - 5\times0 + 2\times4 = 26$	M1A1	3	
(ii)	$S_{-1} = \dfrac{\sum\alpha\beta\gamma}{\alpha\beta\gamma\delta}$	$S_{-1} = \dfrac{-5}{-2} = \dfrac{5}{2}$	M1A1		
	Finds S_3 from formula.	$S_3 = 3\times0 - 5\times4 + 2\times\dfrac{5}{2} = -15$	M1A1		
	Finds S_5 from formula.	$S_5 = 3\times(-15) - 5\times6 + 2\times0 = -75$	M1A1	6	
		$\sum\alpha^2\beta^3 = S_2S_3 - S_5$	M1		
		$= 6\times(-15) - (-75) = -15$	M1A1	3	**[14]**

	OR				
(i)	Reduces **M** to echelon form.	$\begin{pmatrix} 2 & 1 & -1 & 4 \\ 3 & 4 & 6 & 1 \\ -1 & 2 & 8 & -7 \end{pmatrix} \rightarrow \ldots \rightarrow \begin{pmatrix} 2 & 1 & -1 & 4 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	M1A1		
	Uses dimension theorem.	$\text{Dim}(\mathbf{M}) = 4 - 2 = 2$	A1		
(ii)	States basis for R.	Basis for R is $\left\{ \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \right\}$. (OE)	B1		
	Finds cartesian equation for R.	$x = 2\lambda + \mu$ $y = 3\lambda + 4\mu \quad \Rightarrow 2x - y + z = 0$ $z = -\lambda + 2\mu$	M1A1		
(iii)	Finds basis for null space.	$2x + y - z + 4t = 0$ $y + 3z - 2t = 0$ $t = \lambda$ and $z = \mu$ $\Rightarrow y = 2\lambda - 3\mu$ and $x = -3\lambda + 2\mu$	M1		
		$\Rightarrow$ Basis of null space is $\left\{ \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \end{pmatrix} \right\}$ (OE)	A1A1	9	
	Evaluates k.	$2 \times 8 - 7 + k = 0 \Rightarrow k = -9$	B1		
	Finds a particular solution.	$5\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \\ -9 \end{pmatrix}$ (OE) (via equations)	M1A1		
	Finds general solution.	$\mathbf{x} = \begin{pmatrix} 5 \\ -2 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \end{pmatrix}$.	M1A1	5	**[14]**

5 - (9231-S 2013-Paper 1/3-Q2) - *Roots of polynomial equations*

Makes substitution. Squares. Obtains result.	$y^2 - 4y + 3y^{\frac{1}{2}} - 2 = 0$ $\Rightarrow 9y = 4 + y^4 + 16y^2 - 4y^2 + 16y - 8y^3$ (N.B. Must see both terms in y^2.) $\Rightarrow y^4 - 8y^3 + 12y^2 + 7y + 4 = 0$ (AG)	M1 M1 A1	 3	
	$S_2 = 0^2 - 2\times(-4) = 8$ $S_8 = 8S_6 - 12S_4 - 7S_2 - 16$ $\Rightarrow S_8 = 8S_6 - 12S_4 - 56 - 16 = 8S_6 - 12S_4 - 72$ (AG)	B1 M1 A1	 3	 **[6]**
	Alternatively – for final two marks. $S_2 = 8$, $S_3 = -9$, $S_4 = 40$, $S_5 = -60$, $S_6 = 203$, $S_7 = -378$ $S_8 = 1072$ (generated by substitution of roots in equations and summing.) Then $8S_6 - 12S_4 - 72 = 1624 - 480 - 72 = 1072 = S_8$ M1 requires a complete method, A1 if all correct.			

6 - (9231-S 2013-Paper 1/1-Q3) - *Roots of polynomial equations*

Uses $\sum\alpha = \frac{-b}{a}$.	$c = 2$	B1	1	
Uses substitution	$(\alpha + \beta = c - \gamma$ etc.$) \Rightarrow y = c - x \Rightarrow x = c - y$ $(2-y)^3 - 2(2-y)^2 - 3(2-y) + 4 = 0 \ldots$(their c)	M1 M1		
to obtain required cubic equation.	$\Rightarrow y^3 - 4y^2 + y + 2 = 0$	A1	3	
Obtains equation whose roots are reciprocals of those in previous cubic equation.	Uses $z = y^{-1}$ to obtain $2z^3 + z^2 - 4z + 1 = 0$	M1A1	2	
Uses $\sum\alpha^2 = \left(\sum\alpha\right)^2 - 2\sum\alpha\beta$	$\sum\frac{1}{(\alpha+\beta)^2} = \left(\frac{1}{2}\right)^2 - 2(-2) = 4\frac{1}{4}$	M1A1	2	**[8]**

7 - (9231-W 2013-Paper 1/1-Q2) - *Roots of polynomial equations*

(i)	Finds S_2.	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= 0 - 2(-p) = 2p$ (AG)	B1	1	
(ii)	Finds S_3.	$\alpha^3 + \beta^3 + \gamma^3 = p\sum\alpha + 3q = 0 + 3q = 3q$ (AG)	M1A1	2	
(iii)	Finds S_5.	$\alpha^5 + \beta^5 + \gamma^5 = p\sum\alpha^3 + q\sum\alpha^2$ $= p.3q + q.2p = 5pq$ $\Rightarrow 6\sum\alpha^5 = 30\text{pq} = 5\sum\alpha^3\sum\alpha^2$ (AG)	M1 A1 A1	 3	 **[6]**

8 - (9231-W 2013-Paper 1/3-Q5) **-** *Roots of polynomial equations*

Uses sum of roots	$\sum\alpha = 3a = -\frac{36}{8} = -\frac{9}{2} \Rightarrow a = -\frac{3}{2}$	M1A1		
Uses product of roots	$\alpha\beta\gamma = a\left(a^2 - d^2\right) = \frac{21}{8}$	M1		
Substitutes for a,	$\frac{9}{4} - d^2 = -\frac{2}{3} \times \frac{21}{8} = -\frac{7}{4}$	M1		
and solves	$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$	A1		
	Roots are $-\frac{7}{2}, -\frac{3}{2}, \frac{1}{2}$	A1		
Uses sum of products in pairs.	$\sum\alpha\beta = \frac{21}{4} - \frac{7}{4} - \frac{3}{4} = \frac{k}{8}$	M1		
(Or expands $(2x+7)(2x+3)(2x+1)$ and equates coefficients)	$\Rightarrow k = 22$	A1	**(8)**	**[8]**

9 - (9231-S 2014-Paper 1/1-Q1) **-** *Roots of polynomial equations*

Let roots be α, α^{-1} and β $\Rightarrow \alpha + \alpha^{-1} + \beta = 0$ (1) Any of three for M1	M1
Product of roots $\Rightarrow \beta = -q$ (2) A1 for another.	A1
Sum of products in pairs $\Rightarrow 1 + \beta(\alpha + \alpha^{-1}) = p$ (3) A1 for a third	A1
From (1) and (3) $1 - \beta^2 = p$ Wrong sign in (2) scores	M1
Using (2) $1 - q^2 = p$ or $p + q^2 = 1$ M1A0A1M1A0	A1
	[5]

10 - (9231-W 2014-Paper 1/1-Q11) **-** *Roots of polynomial equations, Matrices*

E (i)	$\alpha+\beta+\gamma+\delta=-4$	B1 (1)
(ii)	$\alpha^2+\beta^2+\gamma^2+\delta^2=(-4)^2-2\times 2=12$	M1A1 (2)
(iii)	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{-(-4)}{1}=4$	M1A1 (2)
(iv)	$\frac{\alpha}{\beta\gamma\delta}+\frac{\beta}{\alpha\gamma\delta}+\frac{\gamma}{\alpha\beta\delta}+\frac{\delta}{\alpha\beta\gamma}=\frac{\alpha^2+\beta^2+\gamma^2+\delta^2}{\alpha\beta\gamma\delta}=\frac{12}{1}=12$	M1A1 (2)
	$y=x+1\Rightarrow x=y-1$	
	$(y-1)^4+4(y-1)^3=y^4-6y^2+8y-3$	M1A1
	$2(y-1)^2-4(y-1)+1=2y^2-8y+7$	A1
	$\Rightarrow x^4+4x^3+2x^2-4x+1=y^4-4y^2+4=0$	A1
	$(y^2-2)^2=0\Rightarrow y=\pm\sqrt{2}$ (twice).	A1
	$\Rightarrow x=\pm\sqrt{2}-1$ (twice). (Some indication of four roots for final mark.)	M1A1 (7) **[14]**
O (i)	$\mathbf{Ae}=\lambda\mathbf{e}$; since $\mathbf{A}$ is non-singular $\mathbf{Ae}\neq 0\Rightarrow\lambda\neq 0$ ($\mathbf{e}\neq 0$) .	M1A1 (2)
(ii)	$\mathbf{Ae}=\lambda\mathbf{e}\Rightarrow\mathbf{A}^{-1}\mathbf{Ae}=\mathbf{A}^{-1}\lambda\mathbf{e}$	M1
	$\Rightarrow\mathbf{e}=\lambda\mathbf{A}^{-1}\mathbf{e}\Rightarrow\ \mathbf{A}^{-1}\mathbf{e}=\frac{1}{\lambda}\mathbf{e}$	A1 (2)
	Eigenvalues of **A** are -2 , -1 , 3 (1 mark for any one, 2 marks for all three.)	B2,1,0
	Corresponding eigenvectors are: $\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}2\\1\\0\end{pmatrix},\begin{pmatrix}-6\\25\\20\end{pmatrix}$ (M1A1 for one, A1 for each other.)	M1A1 A1 A1
	(N.B. May come from using eigenvalues of **A** + 3**I**.)	
	$\mathbf{P}=\begin{pmatrix}1&2&-6\\0&1&25\\0&0&20\end{pmatrix}$	B1√
	Eigenvalues of **A** + 3**I** are 1 , 2 , 6 (Award B3 if obtained from **A** + 3**I**.)	B1√
	Eigenvalues of $(\mathbf{A}+3\mathbf{I})^{-1}$ are $1,\frac{1}{2},\frac{1}{6}$	B1√
	$\mathbf{D}=\begin{pmatrix}1&0&0\\0&\frac{1}{2}&0\\0&0&\frac{1}{6}\end{pmatrix}$ (CAO)	B1 (10) **[14]**