A-Level Edexcel

MATHS CORE

UNIT C34(IAL)

2015 — 2019

1 - (6663-S 2015-Unit C34(IAL)-Q2) - *ALGEBRA, BINOMIAL EXPANSION, PARTIAL FRACTIONS*

Given that

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2} \equiv \frac{A}{(1-2x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}$$

(a) find the values of the constants A and C and show that $B = 0$

(4)

(b) Hence, or otherwise, find the series expansion of

$$\frac{4(x^2+6)}{(1-2x)(2+x)^2}, \qquad |x| < \frac{1}{2}$$

in ascending powers of x, up to and including the term in x^2, simplifying each term.

(5)

2 - (6663-S 2016-Unit C34(IAL)-Q6) - *ALGEBRA, GRAPHS*

Given that a and b are constants and that $a > b > 0$

(a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the x-axis and the y-axis.

(5)

(b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b.

(4)

3 - (6663-S 2017-Unit C34(IAL)-Q4) - *ALGEBRA, BINOMIAL EXPANSION*

$$f(x) = \frac{27}{(3-5x)^2} \qquad |x| < \frac{3}{5}$$

(a) Find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(5)

Use your answer to part (a) to find the series expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \dfrac{27}{(3+5x)^2} \qquad |x| < \dfrac{3}{5}$

(1)

(c) $h(x) = \dfrac{27}{(3-x)^2} \qquad |x| < 3$

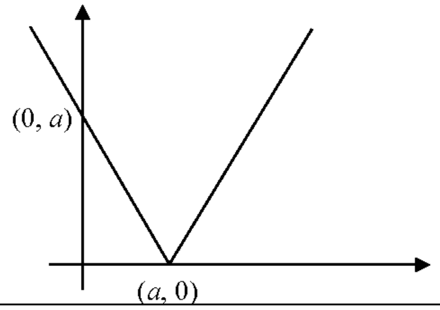
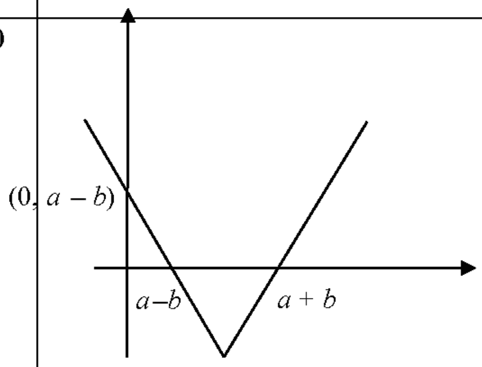
(2)

ANSWERS

1 - (6663-S 2015-Unit C34(IAL)-Q2) **-** *ALGEBRA, BINOMIAL EXPANSION, PARTIAL FRACTIONS*

(a) Way 1	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$ Let $x=-2 \Rightarrow 40 = 5C \Rightarrow C = 8$ Let $x=\frac{1}{2} \Rightarrow 25 = 6.25A \Rightarrow A = 4$ $\quad A=4, C=8$ **Compare constants / terms in *x*** or substitute another value of x into identity and conclude that $B=0$ e.g. $24 = 4A+2B+C \Rightarrow B = 0$ *
Way 2 (a)	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$ Compare x^2 : so $4 = A-2B$, x: so $0 = 4A-3B-2C$, constants: so $24 = 4A+2B+C$ **So** $A=4, C=8$, and $B=0$*
Way 1 (b)	$\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(1-2x)^{-1} + 8(2+x)^{-2} = 4(1-2x)^{-1} + 8\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$ See... $\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ... $\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$ ... $\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ... $\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$ $= 4(1+2x+4x^2+..)+2\left(1-x+\frac{3}{4}x^2\right) \quad = 6+6x+\frac{35}{2}x^2$
Way 2 (b)	Or $\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(x^2+6)\times(1-2x)^{-1}\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$ See... $\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ... $\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$ ... $\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ... $\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$ $= 4(x^2+6)(1+2x+4x^2+..)\times\frac{1}{4}\left(1-x+\frac{3}{4}x^2\right) = 6+6x+\frac{35}{2}x^2$

2 - (6663-S 2016-Unit C34(IAL)-Q6) **-** *ALGEBRA, GRAPHS*

(a)(i)		V shape with vertex on *x*-axis but **not** at the origin.	B1
		Correct V shape with (0, *a*) or just *a* and (*a*, 0) or just *a* marked in the correct places. Left branch must cross or touch the *y*-axis. Allow coordinates the wrong way round if marked in the correct place.	B1
			(2)
(a)(ii)		Their part (i) translated down (by any amount) but clearly not left or right, or the correct shape i.e. a V with the vertex in 4th quadrant.	B1ft
		A *y*-intercept of *a* – *b* on the positive *y*-axis or intercepts of *a* – *b* and *a* + *b* on the positive *x*-axis with *a* + *b* to the right of *a* – *b*	B1
		A fully correct diagram.	B1
			(3)
(b)	$x-a-b=\frac{1}{2}x\Rightarrow x=\ldots$ **or** $-x+a-b=\frac{1}{2}x\Rightarrow x=\ldots$	Solves $x-a-b=\frac{1}{2}x$ **or** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$ Allow $<$ or $>$ for $=$.	M1
	$x-a-b=\frac{1}{2}x\Rightarrow x=\ldots$ **and** $-x+a-b=\frac{1}{2}x\Rightarrow x=\ldots$	Solves $x-a-b=\frac{1}{2}x$ **and** solves $-x+a-b=\frac{1}{2}x$ as far as $x=\ldots$ Allow $<$ or $>$ for $=$.	M1
	$\frac{2}{3}(a-b)<x<2(a+b)$	ddM1: Chooses inside region. A1: Allow alternatives e.g. $x<2(a+b)$ **and** $x>\frac{2}{3}(a-b)$, $x<2(a+b)\cap x>\frac{2}{3}(a-b)$, $\left(\frac{2}{3}(a-b),\ 2(a+b)\right)$ but not $x<2(a+b)$, $x>\frac{2}{3}(a-b)$	ddM1A1
			(4)
			(9 marks)

3 - (6663-S 2017-Unit C34(IAL)-Q4) **-** *ALGEBRA, BINOMIAL EXPANSION*

(a)	$27(3-5x)^{-2} = 27\times\frac{1}{9}\left(1-\frac{5}{3}x\right)^{-2}$	B1, B1
	$=3\left(1+(-2)\left(-\frac{5}{3}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5}{3}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{5}{3}x\right)^3+\ldots\right)$	M1
	$=3+10x+25x^2+\frac{500}{9}x^3+\ldots$	A1, A1
		(5)
(b)	$27(3+5x)^{-2} = 27(3+5x)^{-2}$	
	$=3-10x+25x^2-\frac{500}{9}x^3+\ldots$	B1ft
		(1)
(c)	$27(3-x)^{-2} = 3+\frac{10}{5}x+\frac{25}{5^2}x^2+\frac{500}{9\times5^3}x^3$	M1
	$=3+2x+x^2+\frac{4}{9}x^3$	A1
		(2)
		(8 marks)